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Editorial

POLISH MARITIME RESEARCH is a scientific journal of worldwide circulation. The journal appears as a quarterly four times a year. The first issue of it was published in September 1994. Its main aim is to present original, innovative scientific ideas and Research & Development achievements in the field of:

Engineering, Computing & Technology, Mechanical Engineering,

which could find applications in the broad domain of maritime economy. Hence there are published papers which concern methods of the designing, manufacturing and operating processes of such technical objects and devices as: ships, port equipment, ocean engineering units, underwater vehicles and equipment as well as harbour facilities, with accounting for marine environment protection.

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**Parametric method for evaluating optimal ship deadweight**

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**ABSTRACT**

The paper presents a method of choosing the optimal value of the cargo ships deadweight. The method may be useful at the stage of establishing the main owners requirements concerning the ship design parameters as well as for choosing a proper ship for a given transportation task. The deadweight is determined on the basis of a selected economic measure of the transport effectiveness of ship – the Required Freight Rate (RFR). The mathematical model of the problem is of a deterministic character and the simplifying assumptions are justified for ships operating in the liner trade. The assumptions are so selected that solution of the problem is obtained in analytical closed form. The presented method can be useful for application in the pre-investment ships designing parameters simulation or transportation task studies.

**Keywords**: ship design; owners requirements; optimal deadweight; required freight rate

**INTRODUCTION**

Principles of engineering activity which are accepted in technique and technology should meet requirements of both technical safety and economic efficiency. Criterial measures used for evaluating qualities of engineering investment products have usually a combined financial and time-related nature. The navigating activity performed in the conditions of market competition requires thoroughness in selection of watercraft, for the transportation task to be performed optimally. Realisation of an investment project is connected with heavy capital involvement dedicated to cover the cost of ship building, purchase of a used ship, or ship charter - to bring certain investment gains. Economic and technical analyses discussing the issue of selecting optimal ship parameters to perform a given sea transportation task, [1, 2, 3 and 4] for instance, indicate that the problem analysis should take into account, among other factors, the following market conditions:

- investment goal;
- investment cost;
- cost of capital raising;
- balance of incomes and operating costs;
- risk of demand decrease, inflation phenomenon;
- risk of freight rate decrease, tax regulations;
- required profitability and activity of competitors.

Formulating and solving the above task requires a series of simplifying assumptions and approximate predictions to make the basis for conclusions on the optimal variant of the investment decision. Despite its approximate nature, the procedure can bring better support for decision making than general feeling and intuition.

**STRUCTURE OF OPERATING COSTS**

The operating costs of transport activity of the ship which depend on its deadweight can be divided into:

- direct costs, such as handling charges and movement cost;
- indirect costs connected with ship maintenance.

The direct costs include such items as:

- costs of (heavy and light) fuel, cost of oils and lubricants, harbour and canal dues, tug services, loading and unloading costs, commissions of agents, brokers and others.

The indirect costs concerning ship maintenance include such items as, for instance:

- salaries, social insurance, crew living cost, cost of ship repair and maintenance, non-life insurance costs, ship depreciation cost.

The handling and movement costs depend of sailing conditions, i.e.:

- number of loading and unloading harbours;
- length of the voyage route;
- weather conditions;
- efficiency of ship capacity utilisation (ballast voyages);
- cruising speed of the ship;
− main engine power output and the rate of fuel consumption;
− time of waiting on road;
− harbour dues, broker’s commissions and other direct costs.

An important item in this list is the cost of fuel (bunker), oils, and lubricants, the proportion of which exceeds 50% of variable costs. Their level depends on: the ship deadweight and power output, the main engine type (internal combustion engine, turbine, etc.), the type of fuel and its specific consumption, technical state of the propulsion system, and modernity of the marine power plant equipment.

The handling costs compose the next group of variable costs depending on ship deadweight — i.e. on the volume and type of the transported cargo, and on the handling rates charged by harbour authorities. The harbour and canal dues include wharfage, canal dues, and the costs of pilotage and tug services.

The tonnage maintenance cost includes crew wages and living costs, the costs of ship repair (repair fund allowance) and conservation, in particular the costs of general classification overhauls done every five years and intermediate routine repairs done every year. In a relatively long time period, the cost of ship repair stops is assessed approximately as equal to 10% of the time of its operation.

The next cost is the costs of ship insurance (underwriting) of ‘casco’ type. Along with the capacity, the level of the insurance rate and the total insurance cost are affected by the evaluation of the failure frequency in the fleet operated by the ship owner and the prestige of the institution being the ship classifier. In case of great care taken of the ship and cargo, and failure-free ship operation the insurance rate can be reduced.

The crew wages compose less than twenty percent of the fixed costs — depending on the ship owner and ship flag (cheap flags). They include basic wages, payments for overtimes and doing special tasks, such as contracts for specific tasks for instance, social and health insurance costs, wages of crew being ship owner’s reserve, representation costs and the captain fund. They also include the crew living cost, strongly affected by the number of crew members and law regulations referring to the ship of a given type and dimensions.

The activity of ship owner’s land service forces in the area of operating costs consist in: (1) carrying out effective and flexible canvassing polities in order to increase the volume of the transported cargo and minimise ballast voyages; (2) reducing the level of expenses spent within the framework of transportation tasks, (3) attempting to shorten the time of ship lay time in the harbour and on road, (4) using bonus payments for failure-free operation, rational fuel consumption, damageless loading, and proper ship maintenance.

Before placing an shipbuilding order, the ship owner usually performs some analyses to assess optimal parameters of the ship to be built (formulation of design assumptions), see [4, 7 and 8] for instance. Selecting ship deadweight has both economic and technical aspects, as it remarkably affects the economic results of the entire investment project. When the selected deadweight is larger than that really needed, it results in excessive investment and operating costs, while when it is too small it leads to the loss of some profits due to not fulfilling part of transport demand.

In the below presented method, relevant selection of analytical relations and simplifying assumptions had made the basis for working out a mathematical model of the problem, in a closed analytical form, which can be used for evaluating optimal ship deadweight. The obtained analytical form of the solution makes it possible to evaluate a qualitative impact of model variables on the optimal ship deadweight, and test the effect of market conditions of the efficiency if ship owner’s investment decisions.

**PROBLEM FORMULATION AND ASSUMPTIONS**

A set of basic design assumptions for a ship with the given operating function usually includes such parameters as: deadweight $P_n$, net capacity $P_l$, number of passengers $N$, operating speed $v$, volume of holds $V$, radius of autonomous action $R$, and other quantities of lower importance. Analysed is the design task oriented on evaluating the ship deadweight $P_n$: $\text{− at given speed } v$; $\text{− at given action radius } R$; $\text{− with cargo handling performed in } s$ harbours; $\text{− at investment profitability rate equal to } r$; to arrive at the lowest possible value of the required freight rate $RFR$ in given technical and economic conditions, used as the measure of economic efficiency of the designed ship.

The real cargo supply in harbours has generally the stochastic nature, which in particular refers to tramping. The present method assumes a deterministic model of cargo supply, which takes into account the random nature of cargo supply by using the coefficient $\varepsilon$ which expresses the average utilisation of ship net capacity in the voyage. In a relatively long time interval this coefficient estimates the averaged real cargo supply. The advantage of the proposed approach is easy calculation of this coefficient based on the records in logs of other ships operating on the analysed shipping lane.

The assumed criterial measure for general evaluation of the quality of the designed ship is the minimal value of the required freight rate $RFR$, which results from z reasons in [3, 5, 6], for instance. This coefficient determines the economic efficiency of the investment project and represents the lowest freight rate which the ship owner has to get to arrive at the assumed profitability rate $r$ at given investment and operating costs, and at the assumption that the ship will be in service in $Z$ days per year during $m$ years.

Adopting the minimal $RFR$ rate as the criterial measure in evaluating the ship deadweight is justified by the fact that for future real freight rates in force on a given shipping lane the highest profitability will be obtained by the ship having the lowest required freight rate. If the future real freight rates are higher than the minimal freight rate $RFR$, then the real profitability rate will be higher than the assumed $r$. In case the real freight rates turn out lower than the calculated $RFR$ value, the investment project will not bring the assumed profitability $r$.  

**OPTIMISATION OF SHIP PARAMETERS**

In technique and technology, optimisation consists in selecting a permissible solution which is the best in the sense of the assumed measure (criterion) of task evaluation. In shipbuilding and navigation the need for optimisation is observed in two activity areas:

− operating activity — where it consists in selecting ship parameters which are optimal in the sense of transportation task realisation evaluation;
− designing activity — where it consists in selecting parameters of the designed ship which are optimal in the sense of the adopted criterion of ship quality evaluation.
MATHEMATICAL MODEL

In the presented method of evaluating optimal ship deadweight, the parameters of crucial importance for the mathematical model are: the operating speed of the ship, the capacity of cargo handling utilities, the number of cargo handling harbours, and the time of waiting for handling operations. The cost of building a ship planned to sail at a steady speed depends on its deadweight \( P_n \) and, according to \([1, 2, 3]\), it increases more slowly than the linear function. Consequently, the predicted investment cost (the ship cost) \( J \) can be approximated by the relation:

\[
J = K_j \cdot P_n^{2/3} \quad \Rightarrow \quad K_j = J_o \cdot P_n^{-2/3}
\]

where the proportionality coefficient \( K_j \) can be calculated based on prices \( J_o \) and deadweight values \( P_n \) of similar ships.

The annual operating cost \( AOC \) of the ship which depends on ship deadweight mainly refers to the cost of the consumed fuel and the cost of handling operations. The remaining components of the operating cost, such as crew wages, for instance, can be omitted assuming that their level does not depend, or only slightly depends, on ship deadweight.

The average annual costs of the lubricating oil and repair were taken into account in the coefficient \( \mu > 1 \) which increases the cost of fuel. At these assumptions the annual operating cost \( AOC \) of the ship propulsion is:

\[
AOC = \mu \cdot n \cdot T_M \cdot C_j \cdot G_j \cdot N_e
\]

The power \( N_e \) can be expressed using the admiralty formula, then:

\[
AOC = \mu \cdot n \cdot T_M \cdot C_j \cdot G_j \cdot \frac{D^{2/3} \cdot v^3}{Ca} = \mu \cdot n \cdot T_M \cdot C_j \cdot G_j \cdot \left[ \frac{\varepsilon \cdot \lambda \cdot P_n}{\eta} \right] \cdot \frac{v^3}{Ca} = K_c \cdot n \cdot P_n^{2/3}
\]

where:
- \( D \) – current ship displacement,
- \( n \) – number of voyages per year,
- \( T_M \) – time of one voyage,
- \( C_j \) – unit fuel price, expressed in \( \$/t \), for instance,
- \( G_j \) – specific fuel consumption, expressed for instance in \( \text{g/kWh} \),
- \( \lambda \) – deadweight efficiency,
- \( \varepsilon \) – net capacity efficiency,
- \( \eta \) – deadweight-displacement coefficient,
- \( Ca \) – admiralty constant.

The coefficients:

\[
\lambda = \frac{P_n - Z}{P_n} \quad \eta = \frac{P_n}{D}
\]

are assumed, or calculated based on data from similar ships. Here \( Z \) represents the mass of fuel reserve in one voyage. The factor \( K_c \) is equal to:

\[
K_c = \frac{\mu \cdot R \cdot C_j \cdot G_j \cdot \left[ \frac{\varepsilon \cdot \lambda}{\eta} \right] \cdot \frac{v^3}{Ca}}{C_j \cdot G_j \cdot \left[ \frac{\varepsilon \cdot \lambda}{\eta} \right] \cdot \frac{v^3}{Ca}} = \frac{\mu \cdot C_j \cdot G_j \cdot v^2 \cdot R \cdot \left( \frac{\varepsilon \cdot \lambda}{\eta} \right)^{2/3}}{Ca}
\]

The level of the annual handling cost \( AHC \) depends on the mass of cargo, the number of voyages per year \( n \) and the unit handling rate \( W_j \):\[
AHC = n \cdot (2 \cdot \varepsilon \cdot \lambda \cdot P_n) \cdot W_j = Kh \cdot n \cdot P_n
\]

The parameter \( Kh \) represents:

\[
Kh = 2 \cdot \varepsilon \cdot \lambda \cdot W_j
\]

The time of voyage \( T \) is composed of the sailing time \( T_s \), the time of waiting on road and in harbour \( T_o \), and the time of cargo loading and unloading \( T_Q \) when the capacity of the handling facilities is \( Q \):

\[
T = T_M + T_O + T_Q = \frac{R}{v} + T_O + 2 \cdot \frac{P_l}{Q} = \frac{R}{v} + T_O + 2 \cdot \varepsilon \cdot \lambda \cdot P_n
\]

The number of ship voyages per year depends on the time of ship operation in the entire year \( Z_h \) and the time of one voyage \( T \):

\[
n = \frac{Z_h}{T} = \frac{Z_h}{\frac{R}{v} + T_O + 2 \cdot \varepsilon \cdot \lambda \cdot P_n} = \frac{K_q}{R \cdot Q + T_O \cdot v \cdot Q + 2 \cdot \varepsilon \cdot \lambda \cdot P_n} = \frac{K_r}{K_p \cdot P_n}
\]

where:
- \( K_q = Z_h \cdot v \cdot Q \)
- \( K_r = R \cdot Q + T_O \cdot v \cdot Q \)
- \( K_p = 2 \cdot \varepsilon \cdot \lambda \cdot v \)

The annual transportability of the ship is:

\[
ACC = n \cdot \varepsilon \cdot \lambda \cdot P_n
\]

The discounted financial balance of the investment project is:

\[
\frac{AAC}{CRF(r, m)} = J + \frac{AOC + AHC}{CRF(r, m)}
\]

where \( AAC \) represents the discounted annual average cost, while \( CRF \) stands for the capital recovery factor:

\[
CRF(r, m) = \frac{r}{1 - (1 + r)^{-m}}
\]

Taking into account the inflation rate \( i \) and the tax rate \( t \), the discounted financial balance of the investment project is given by the relations:

\[
\frac{AAC}{CRFT(r, m, i, t)} = J + \frac{AOC + AHC}{CRFT(r, m, i, t)}
\]

The discounted annual average cost \( AAC \) of the investment project is:

\[
AAC(P_n) = J(P_n) \cdot CRFT(r, m, i, t) + AOC(P_n) + AHC(P_n)
\]
Taking into account other costs is justified when they are affected by ship deadweight. In that case they should be summed up:

$$AAC(P_n) = J(P_n) \cdot CRFT(r, m, i, t) \quad +$$
$$+ AHC(P_n) + \sum AOC_i(P_n)$$

(17)

**DEADWEIGHT MINIMISING THE FREIGHT RATE RFR**

If the cost AAC compensates the annual fixed freight incomes, then the rate at which this equality takes place will secure the investment efficiency rate \( r \) in the formulas for CRF and CRFT. The freight rate \( RFR \) is defined by the ratio of the annual freight incomes corresponding to AAC to the annual transportability ACC of the ship

$$RFR = \frac{AAC}{ACC} = \frac{J \cdot CRFT + AOC + AHC}{ACC} =$$

$$= \frac{K_j \cdot CRFT \cdot Pn^{2/3} + K_c \cdot n \cdot Pn^{2/3} + K_h \cdot n \cdot Pn}{\epsilon \cdot \lambda \cdot Pn}$$

(18)

Transforming the above formula to the form explicitly dependent on deadweight we get:

$$RFR = \frac{Pn^{-1/3} \left( \frac{K_j \cdot CRFT}{n} + K_c \right) + \frac{K_h}{\epsilon \cdot \lambda}}{\epsilon \cdot \lambda}$$

(19)

$$= \frac{Pn^{-1/3}}{\epsilon \cdot \lambda} \left( \frac{K_j \cdot CRFT}{n} + K_c \right) + \frac{K_h}{\epsilon \cdot \lambda}$$

The lowest freight rate \( RFR \) corresponds to the ship deadweight \( Pn \) which meets the necessary condition for the existence of the extremum of the function \( RFR \):

$$\frac{\partial RFR}{\partial Pn} = 0$$

(21)

This condition takes the form:

$$\frac{\partial RFR}{\partial Pn} =$$

$$= -1 \cdot \frac{Pn^{-4/3}}{3 \epsilon \cdot \lambda} \left( \frac{K_j \cdot CRFT \cdot (K_r + K_p \cdot Pn)}{K_q} + K_c \right) +$$

$$+ \frac{Pn^{-1/3}}{\epsilon \cdot \lambda} \frac{K_j \cdot CRFT \cdot K_p}{K_q} = 0$$

(22)

After multiplying both sides by the indicated parameter:

$$-\frac{1}{3} \cdot \frac{Pn^{-4/3}}{\epsilon \cdot \lambda} \left( \frac{K_j \cdot CRFT \cdot (K_r + K_p \cdot Pn)}{K_q} + K_c \right) +$$

$$+ \frac{Pn^{-1/3}}{\epsilon \cdot \lambda} \frac{K_j \cdot CRFT \cdot K_p}{K_q} = 0$$

(23)

and ordering the terms, the equation takes the form:

$$\frac{K_j \cdot CRFT \cdot (K_r + K_p \cdot Pn)}{K_q} +$$

$$+ \frac{K_p \cdot CRFT \cdot K_p}{K_q} = 0$$

(24)

And after multiplying again by the indicated parameter:

$$\frac{K_j \cdot K_p \cdot CRFT \cdot Pn}{K_q} +$$

$$+ \frac{K_p}{2 \cdot K_j \cdot CRFT} = 0$$

(25)

we can determine the deadweight \( Pn \), at which the required freight rate \( RFR \) is the lowest:

$$Pn = \frac{1}{2} \left( \frac{K_r \cdot K_p^{-1} + K_c \cdot K_j^{-1} \cdot K_p^{-1} \cdot CRFT^{-1}}{K_q} \right)$$

(26)

Within the framework of the analysed model, the determined ship deadweight is the optimal deadweight represented by the ship with the minimal freight rate which can be expressed explicitly using the adopted variables of the mathematical model:

$$P_n = \frac{1}{2} \left( K_r + \frac{K_c \cdot K_j}{K_p \cdot CRFT} \right) = \frac{Q}{4 \cdot \epsilon \cdot \lambda \cdot v}$$

(27)

$$\sum R + T_o \cdot v +$$

$$+ \frac{\mu \cdot C_i \cdot G_j \cdot v^3 \cdot R \cdot (\epsilon \cdot \lambda \cdot P_{n_0})^{2/3} \cdot Z_h}{J_o \cdot C_a \cdot \eta^{2/3} / (r + i + r \cdot i)}$$

$$\left( \frac{1}{1 + (1 + r + i + r \cdot i)^{-m} \cdot (1 - t)} \right)$$

The above relation can be applied in simulation analyses of the effect of individual model parameters on optimal ship deadweight.

**SAMPLE APPLICATION OF THE METHOD**

The sample application of the method refers to the optimal deadweight calculations for a cargo ship sailing at a speed equal to 17.5 kn when the assumed action radius is equal to 8000 Mm. The calculations took into account technical and economic conditions of ship building and operation defined by the parameters given in Table 1 and the profitability rate \( r \) required by the ship owner.

The assumed criterion for deadweight evaluation was the minimal freight rate which secures the required profitability of the investment project. This optimal deadweight corresponds to the minimal \( RFR \) value. If the ship deadweight differs from the optimal value, the freight rate which secures profitability is higher.
Tab. 1. Sample evaluation of optimal ship deadweight

<table>
<thead>
<tr>
<th>Ship owner’s assumptions</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumed ship speed</td>
<td>v</td>
<td>17.5</td>
<td>[kn]</td>
</tr>
<tr>
<td>Autonomous action radius</td>
<td>Ra</td>
<td>8000</td>
<td>[Mm]</td>
</tr>
<tr>
<td>Average annual inflation rate</td>
<td>i</td>
<td>0.03</td>
<td>[-]</td>
</tr>
<tr>
<td>Number of ship operation days per year</td>
<td>Zd</td>
<td>340</td>
<td>[days]</td>
</tr>
<tr>
<td>Time of ship operation in hours</td>
<td>Zh</td>
<td>8160</td>
<td>[h]</td>
</tr>
<tr>
<td>Required net profitability rate</td>
<td>r</td>
<td>0.09</td>
<td>[-]</td>
</tr>
<tr>
<td>Income tax rate</td>
<td>t</td>
<td>0.19</td>
<td>[-]</td>
</tr>
<tr>
<td>Number of years of ship operation</td>
<td>m</td>
<td>20</td>
<td>[-]</td>
</tr>
<tr>
<td>Unit handling cost</td>
<td>Wj</td>
<td>5</td>
<td>[$/t]</td>
</tr>
<tr>
<td>Specific fuel consumption</td>
<td>Gj</td>
<td>160</td>
<td>[g/kWh]</td>
</tr>
<tr>
<td>Fuel price</td>
<td>Cj</td>
<td>600</td>
<td>[$/t]</td>
</tr>
<tr>
<td>Handling capacity</td>
<td>Q</td>
<td>50</td>
<td>[t/h]</td>
</tr>
<tr>
<td>Time of waiting on road and in harbour</td>
<td>To</td>
<td>2</td>
<td>[days/voyage]</td>
</tr>
<tr>
<td>Time of waiting on road and in harbour</td>
<td>Toh</td>
<td>48</td>
<td>[h/voyage]</td>
</tr>
</tbody>
</table>

| Parameters of similar ship                      |        |        |        |
| Deadweight of similar ship                      | Pp     | 10532  | [t]    |
| Speed of similar ship                           | v      | 16.5   | [kn]   |
| Displacement of similar ship                    | D      | 14946  | [t]    |
| Engine power of similar ship                    | Ne     | 5741   | [kW]   |
| Admiralty coefficient                           | Ca     | 566    | [-]    |
| Price of similar ship                           | J      | 40 000 000 | [$] |
| Deadweight-displacement coefficient             | Eta    | 0.705  | [-]    |

| Assumed model parameters                        |        |        |        |
| Deadweight efficiency                           | lamb   | 0.9    | [-]    |
| Net capacity efficiency                         | eps    | 0.9    | [-]    |
| Service cost coefficient                        | mi     | 1.1    | [-]    |

| Auxiliary model parameters                      |        |        |        |
| Building cost coefficient                       | Kj     | 83 250 | [-]    |
| Operating cost coefficient                      | Kc     | 501    | [-]    |
| Handling cost coefficient                       | Kh     | 8      | [-]    |
| Cost coefficient                                | Kq     | 7 140 000 | [-] |
| Cost coefficient                                | Kp     | 28     | [-]    |
| Cost coefficient                                | Kr     | 442 000 | [-]    |
| Capital return factor                           | CRF    | 0.148  | [-]    |
| Tax correction CRF                              | CRFT   | 0.182  | [-]    |

| Calculated technical ship parameters            |        |        |        |
| Optimal ship deadweight                         | Pnopt  | 11950  | [t]    |
| Ship displacement                               | Displ  | 16959  | [t]    |
| Engine power                                    | Power  | 5949   | [kW]   |
| Time of 1 voyage                                | Tr     | 892    | [h]    |
| Time of cruising                                | Tm     | 457    | [h]    |
| Time of handling                                | Tq     | 387    | [h]    |
| Number of voyages per year                      | LRR    | 9.1    | [-]    |
| Fuel consumption in 1 voyage                    | ZPR    | 435    | [t]    |

| Calculated economic ship parameters             |        |        |        |
| Required Freight Rate                           | RFR    | 126.8  | [$/t]  |
| Capital Recovery Period                         | Time   | 6.8    | [lat]  |
| Invest Cost                                     | Price  | 43 515 053 | [$] |
| Annual Cargo Capacity                           | ACC    | 88 517 | [t]    |
| Annual Cargo Freight                            | ACF    | 11 220 977 | [$] |
| Annual Fuel Cost                                | AFC    | 2 387 241 | [$]    |
| Annual Cargo Handling Cost                      | AHC    | 885 173 | [$]    |
| Annual Operating Cost                           | AOC    | 3 272 414 | [$]    |
| Average Annual Cost                             | AAC    | 12 097 673 | [$] |
SUMMARY

The paper presents a method for selecting the optimal deadweight of a cargo ship. The method may be useful at the stage of establishing basic ship owner’s requirements concerning ship design parameters, along with choosing a proper ship for a given transportation task. The deadweight is determined on the basis of a selected economic measure of ship’s transport efficiency, which is the Required Freight Rate (RFR). The mathematical model of the problem is of deterministic nature. The adopted simplifying assumptions base on the data obtained for ships operating in the liner trade. The assumptions have been selected in such a way that the solution of the problem is obtained in a closed analytical form. The reported method can be used for calculating pre-investment ships design parameters, or in transportation task studies.

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Resistance Prediction for Hard Chine Hulls in the Pre-Planing Regime

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ABSTRACT

A mathematical representation of calm-water resistance for contemporary planing hull forms based on the USCG and TUNS Series is presented. Regression analysis and artificial neural network (ANN) techniques are used to establish, respectively, Simple and Complex mathematical models. For the Simple model, resistance is the dependent variable (actually \(R/\Delta\) for standard displacement of \(\Delta = 100000\) lb), while the Froude number based on volume (\(F_{nV}\)) and slenderness ratio (\(L/V^{1/3}\)) are the independent variables. In addition to these, Complex model’s independent variables are the length beam ratio (\(L/B\)), the position of longitudinal centre of gravity (\(LCG/L\)) and the deadrise angle (\(\beta\)). The speed range corresponding to \(F_{nV}\) values between 0.6 and 3.5 is analyzed. The Simple model can be used in the concept design phases, while the Complex one might be used for various numerical towing tank performance predictions during all design phases, as appropriate.

Keywords: planing craft; hard chine hulls; resistance evaluation; Artificial-Neural-Network (ANN), TUNS Series, USCG Series; pre-planing regime

INTRODUCTION

Planing craft are by far the most common high speed marine vehicles that enable speeds considerably higher than those of displacement or semi-displacement type of vessels. Planing hulls have smaller resistance for speeds above \(F_{nL} = 0.4, F_{nB} = 0.5\) or \(F_{nV} = 1\). For \(F_{nL}, F_{nB}\) and \(F_{nV}\) around 1, 1.5 and 3÷3.5 respectively, full planing is achieved when dominant hydrodynamic forces carry almost the whole weight of a vessel, while the buoyancy is relatively negligible. Note however that the vessel has to pass through three different regimes – displacement, semi-displacement and planing. To achieve this, the planing hull form, although simple, has certain peculiarities. Notably, it has pronounced hard chine (to enable flow separation), wide transom and straight stern buttock lines. Consequently, the planing hull-form and hull-loading parameters also differ from those used for conventional displacement vessels.

Planing craft resistance tests are rare, albeit needed, since misjudgements made in the early design phases may result in disappointing performance – something that is not so pronounced for the displacement vessels. Grouped calm-water resistance data obtained through the systematic model tests available in the public domain are:

a) Series EMB 50 and Russian Series BK and MBK – both obsolete

b) Series 62 (Clement and Blount 1963) together with its modifications for higher deadrise (Keuning et al 1982 and 1993) set the new standards in planing hull design

c) Series 65 was developed primarily for the hydrofoils, while recent Series SOTON (Taunton et al 2010) was mainly envisaged for higher speeds and comparison with the stepped hulls

d) Series TUNS (Delgado-Saldivar 1993) and USCG (Kowalshyn and Metclaf 2006) – both appropriate for contemporary planing hulls.

Consequently, the data from the Series TUNS (Technical University of Nova Scotia) and USCG (United States Coast Guard) are used here to develop new mathematical models for resistance evaluation. The application zone for all planing hull series mentioned, as well as for the proposed mathematical model developed here, is shown in Fig. 1.

The calm-water resistance needed for power evaluation may be evaluated:

- directly from model tests (still regarded the most reliable)
- using Various mathematical models (usually based on the model tests)

Since model tests are expensive, and CFD-based models are not yet sufficiently reliable or available for every day engineering applications, mathematical models are indispensable. Two main types of mathematical models are used:

Empirical Models – amongst them Savitsky (1964) is by far the most frequently used.

The Savitsky model (initially derived for prismatic hulls only) was improved a few times (Savitsky 2006 and 2012, Blount and Fox 1976) and is mainly intended for pure planing regime, i.e. above $F_{nV} = 3$. The regression models, however, are used for a given hull form and in the speed range for which they have been derived.

A reliable mathematical model for resistance evaluation for the challenging speed range – the hump speeds – of up to $F_{nV} = 3$ or so, is clearly required. For higher speeds the Savitsky method seems to be reliable enough. Consequently, two speed-dependent math models are developed:

- **Simple Model** – developed through the application of regression analysis and based on only two dominant high-speed parameters, i.e. the slenderness ratio $L/V^{1/3}$ and $F_{nV}$, and
- **Complex Model** – developed through the application of Artificial Neural Network (ANN) and based on five principal high-speed parameters, i.e. the slenderness ratio $L/V^{1/3}$ and $F_{nV}$, but also the length beam ratio $L/B$, the position of longitudinal centre of gravity $LCG/L$ and the deadrise angle $\beta$.

Both models are based on the contemporary planing hull forms, i.e. that of TUNS and USCG Series.

**Hull form parameters**

**Ratio of length to beam** is one of the hull form parameters that should obviously be used. According to Clement (1957) and the experience with the Series 62, it is obvious that the projected chine length $L_p$ is better hull representative than other length measures ($L_{WL}$ or $L_{LOA}$ for instance). $L_p$ will be denoted as $L$ from now on. The choice of Beam metric, however, is not so evident. In the past, the mean chine beam $B_{ch}$, the chine beam at transom $B_{PT}$, the maximum chine beam $B_{PXM}$, the chine beam at $LCG$ etc., were all used as effective beam metrics. Taking into account the hull form of TUNS and USCG series ($B_{PXM}$ is equal or almost equal to $B_{PT}$) $B_{PXM}$ is selected here, and is denoted just $B$. Consequently, $L_p/B_{PXM} = L/B$.

**Deadrise angle of hull bottom** is also not standardized, as the representative $\beta$ might be at $L_p/2$, i.e. $\beta_{L_p/2}$, at transom ($\beta_T$), midway ($\beta_{L_p/2} + \beta_T/2$), at $70\%$ of $L_p$ forward of transom (having Series 62 in mind), at $LCG$ ($\beta_{LCG}$), at mean wetted length (having warped prismatic hull for the Savitsky method) etc. Here, the effective deadrise angle at $B_{PXM}$, i.e. $\beta_{B_{PXM}} = \beta$ is selected. Note that deadrise is not given in the dimensionless form.

**Other hull form characteristics**, such as the longitudinal curvature of the hull bottom (shape of buttock lines), the longitudinal distribution of chine beam, the type of sections etc. are all assumed to be similar to that of TUNS and USCG series, i.e. the mathematical model is valid for hull forms that are similar to those of TUNS and USCG models (see MacPherson 1996).

**Hull loading parameters**

**Hull loading** can be represented through the slenderness ratio – $L/V^{1/3}$, the planing area coefficient – $A_p/V^{2.5}$ or the beam-loading coefficient – $V/B_{PXM}^{3/2}$ (used in the Savitsky method). The slenderness ratio was chosen to be the significant hull loading parameter, since the other two parameters are better for higher planing speeds.

**Longitudinal Centre of Gravity $LCG$** is modelled here through the ratio $LCG/L_p = LCG/L_{LOA}$ relative to the transom. $LCG$ is often presented as the distance of the CG from the centroid of area $A_p$ as percentage of $L_p$.

**Performance characteristics**

Choice of an adequate dimensionless speed parameter – the Froude number – is also of primary importance. Three Froude numbers are normally used, i.e. those based on volume
The goal is to extend the applicability of the well known USCG Series consisting of only 4 models, with much broader but less well known TUNS Series consisting of 9 models. In a way, these two similar series should form a new series that would be applicable to contemporary planing hull forms. A similar approach was used in Hubble (1974) and Radojcic (1985). The database of this new series is a starting point for establishment of the mathematical models for resistance evaluation. The database, and hence the subsequent mathematical models, are applicable for the displacement of 100000 lb in the sea water \( \rho = 1026 \text{ kg/m}^3 \), temperature 15°C, viscosity \( \nu = 1.1907 \times 10^{-6} \text{ m}^2 / \text{s} \) and ITTC-1957 friction coefficients with \( C_f = 0 \).

Note however that although the hull shape of both series is relatively similar, the facilities where the series were tested and accordingly the size of the models belonging to each series are completely different. Specifically, the USCG Series was tested in one of the world largest model basins, while the TUNS experiments were performed in a small university basin only 27 m long. Consequently, the USCG models were much larger, weighting between 135 and 220 kg, while the TUNS models weighted between 1 and 3.5 kg only. There is no doubt, therefore, which results are more reliable, and therefore all USCG data were weighted by a factor of 2. See Morabito and Snodgrass (2012) regarding the usefulness of small models, which may be the weakest point in this work.

More about each series can be found in original publications (Delgado-Saldivar 1993 and Kowalysyn & Metcalf 2006). Table 1 tabulates the ranges of hull form and loading parameters for both series.

**Comparison of USCG and TUNS performance characteristics**

A comparison of performance characteristics in the overlapping zone (i.e. for hull and loading parameters that are the same for TUNS and USCG models) follows. As expected the performance characteristics (in this case the dynamic trim \( \tau \) and the resistance-to-weight ratio \( R/\Delta \)) for both series
should be the same. Direct comparison of the performance characteristics is not possible since the models of each series had slightly different hull and loading parameters. Nevertheless, appropriate interpolations preformed within each series enable comparison. Typical results are shown in Figs. 4 and 5 for $\tau$ and $R/\Delta$ respectively.

In the overlapping zone, the $\tau$ values disagree to a great extent regardless of the kind of interpolation used. The $R/\Delta$ values, however, agree quite well. Generally, TUNS hump and below-hump values of $R/\Delta$ are slightly higher (than those of USCG). The depicted diagrams (and many more) are very convincing, so the development of a mathematical model for
evaluation of τ was abandoned, while the development of R/Δ mathematical model continued.

The τ values are obviously important, as τ and R/Δ mirror each other. This does not have to be the case with mathematical models based on model experiments. The Savitsky empirical model, mainly for planing speeds, is even dependent on τ. It should be noted that disagreements between τ_{TUNS} and τ_{USCG} seem to be systematic but rationalization of these disagreements, however, is beyond the scope of this paper.

**Database for development of mathematical models for R/Δ evaluation**

The database needed for the development of the mathematical model for R/Δ evaluation (for Δ = 100000 lb) can be generated on the conclusions reached so far. Few additional comments are needed:

- For TUNS models, the wetted length was calculated as L_{P} = L_{OA}·0.912, while for the USCG models it was only L_{K} (L_{C} was not available).
- For the USCG models, the representative β = β_{Bpx} was taken to be 18 and 21 degrees (at Station 7.5), and not 20 and 23 degrees (often stated values) which is at Station 6.
- For the USCG models L_{p} was estimated from the lines plan, while for the TUNS it was calculated as L_{p} = L_{OA}·0.912 (as suggested in the original Report).

All R/Δ = f(F_{nV}) curves that were taken into account are shown in Fig. 6. Note that the final F_{nV} range is narrower and is between 0.6 and 3.5.

![Fig. 6. R/Δ = f(F_{nV}) curves taken into account for forming the database](image)

**ON APPLICATION OF ARTIFICIAL NEURAL NETWORK AND REGRESSION ANALYSIS**

Two methods for development of mathematical models for resistance (R/Δ) evaluation were applied:

- Regression analysis for development of a Simple mathematical model
- Artificial Neural Networks (ANN) for development of a Complex model.

In this section these two methods will be briefly compared from the mathematical model-maker’s viewpoint.

When the regression analysis is applied, the independent variables consist of two sets of input data: a) Basic independent variables (hull parameters in this case), and b) Various powers and cross-products of powers of basic independent variables. Hence the initial polynomial equation can easily have 100 or more terms, although the number of basic parameters are usually around 5. If some hull’s characteristic is not presented directly through the basic hull parameters, it will be represented indirectly through one of many polynomial terms that appear in the initial equation. Then, by applying a step-by-step procedure and statistical analysis, a best subset is chosen and less significant variables are rejected, ending with finally adopted equation which has considerably less independent variables; of the order of 10 to 20.

In contrast to the regression analysis, with the ANN method, more attention is paid to selection of independent variables. Specifically, the independent variables should be carefully chosen at the very beginning, because the final model will be based on the selected input parameters which form the input layers (X_{k}) for ANN. Incorrectly selected independent variables could result in an erroneous mathematical model, i.e. dependent variable (R/Δ) may be insensitive to the variations of a wrongly selected input variable. On the other hand, if a larger than necessary number of independent variables is assumed, validation of model stability becomes considerably more complex.

The artificial neural network which is used here (Rojas 1996, Miljkovic 2003, Zurek 2007) is of a feed-forward type with a back-propagation algorithm. The network can be expanded up to eight layers. Three types of activation functions, in which data are processed within the neurons, could be chosen: linear function, sigmoid function and hyperbolic tangent function. Sigmoid and hyperbolic tangent functions were thoroughly tested and the sigmoid function (sig = 1/(1 + e^{-x}) was finally adopted since it produced better results. Both, the number of activation functions (3) and the number of layers (8), are limitations of the program that was used – aNETka 2.0 (see Zurek 2007).

Selection of the number of layers and number of neurons within each layer is very important, since when the equation becomes too complex instability might occur. Once the network configuration is adopted, the number of polynomial terms of a model can be pre-determined from the following expression (Simic 2012):

\[
bc = \sum_{i=1}^{N-1} [(a_{i} + 1) \cdot a_{i+1}] + 2(a_{1} + a_{N})
\]

where N represents the number of layers in the network and a_{i} is the number of neurons in each layer. So, for the mathematical model used here for evaluation of R/Δ (see below), the network is composed of five layers with configuration 5-7-5-3-1 (see Fig. 7), and the number of polynomial terms is:

\[
bc = [(a_{i} + 1) \cdot a_{2}] + [(a_{2} + 1) \cdot a_{3}] + [(a_{3} + 1) \cdot a_{4}] +
+ [(a_{4} + 1) \cdot a_{5}] + 2(a_{1} + a_{N}) = [(5 + 1) \cdot 7] +
+ [(7 + 1) \cdot 5] + [(5 + 1) \cdot 3] +
+ [(3 + 1) \cdot 1] + 2(5 + 1) = 116
\]

Obviously, in order to find a ‘good’ solution, whether assessed in terms of accuracy, reliability or applicability, the number of layers and number of neurons in each layer has to be chosen by the model-maker in advance.

It should be noted that it is common practice to omit from the database the data that is intended to be used for verification of the reliability of the network. Consistent with the prior experience (for instance Simic 2012), the entire database was used for training of the algorithm, but considerable effort was invested in checking the reliability and stability of the derived mathematical model.
The objective is to plot $R/\Delta = f(L/V^{1/3}, F_n V)$ relationship, based on TUNS and USCG data, and to compare this with the well known diagram based on the Series 62 and 65, published in Hubble (1974), see Figure 8.

This was done using the regression analysis options in Microsoft’s Excel. The procedure, also used in Radojcic (1997), is the following:

a) Speed-dependent equations (with the same variables for all slenderness ratios) are developed first. Namely, the data having the same $L/V^{1/3}$ were grouped, regardless of the other hull form and loading parameters. 16 groups were formed. Then a trend line $R/\Delta = f(F_n V)$ was produced (a cubic parabola $y = A \cdot x^3 + B \cdot x^2 + C \cdot x + D$) for each group. Figure 9 shows the results for $L/V^{1/3} = 3.90; 5.46$ and 6.87.

b) A second regression analysis is then performed with the regression coefficients cross-faired against the slenderness ratio. So, four new diagrams are then formed, one for each coefficient, i.e. $A, B, C, D = f(L/V^{1/3})$. See Figure 10.

Thus, the first step develops speed dependent equations for discrete $L/V^{1/3}$ values. The second step extracts $F_n V$ and $L/V^{1/3}$ dependent equations – represented as a surface $R/\Delta = A \cdot F_n V^3 + B \cdot F_n V^2 + C \cdot F_n V + D$, as shown in Figs. 11 and 12. The diagram in Fig. 11 is obviously comparable to the one in Fig. 8.

A similar procedure is repeated for $S/V^{2/3}$ and $L_k/L$ – as needed for evaluation of $R/\Delta$ for displacements other than 100000 lb. See resulting Figs. 13 to 16.

Given the scattered initial data and coefficients, as shown in Figs. 9 and 10 respectively, it is amazing that exceptionally nice diagrams can be produced using a simple procedures available in Excel. Power of statistics and regression is also confirmed.
between the vessel length and weight is desired. Nevertheless, users are reminded that the Simple model is a single parameter formulation and that resistance predictions are of reduced quality when hull parameter boundaries other than that of slenderness ratio are approached.

Coefficients A, B, C and D for evaluation of \( R/\Delta \), \( S/V^{2/3} \) and \( L_K/L \) are given in Appendix 1.
Complex mathematical model for R/Δ evaluation

ANN possibilities, procedures etc. were explained in the previous section. This section explains the development of the Complex mathematical model (vs. the Simple Model previously explained) for the evaluation of:

- \( R/\Delta = f(L/V^{1/3}, F_n/V, L/B, LCG/L, \beta) \)
- \( S/V^{2/3} = f(L/V^{1/3}, F_n/V, L/B, LCG/L) \)
- \( L_K/L = f(F_n/V, LCG/L) \)

The last two variables are obviously simpler, so the procedure for the R/Δ model is explained in more details, but the results for all three variables are given.

Several mathematical models for R/Δ were derived and tested, see Table 2. Considerable time was spent for stability checks of the derived models (oscillations in results for values between data points used for the mathematical model derivation). Several derived models with several hundred terms produced good results but were rejected as too complex and impractical for a user/designer. The finally adopted model with 116 polynomial terms follows:

\[
Y = \frac{\text{sig} \left( d_v + \sum_{w=1}^{3} D_w \cdot \text{sig} \left( c_w + \sum_{i=1}^{5} C_w \cdot \text{sig} \left( b_i + \sum_{j=1}^{7} B_j \cdot \text{sig} \left( a_j + \sum_{k=1}^{5} (A_{kj} \cdot (P_k \cdot X_k + R_k)) \right) \right) \right) \right) - G}{H}
\]

where:
- \( X_K = \{L/B, L/V^{1/3}, F_n/V, LCG/L, \beta\} \)
- \( Y = R/\Delta \)
- \( G, H, P_k, R_k, A_{kj}, B_j, C_w, D_w, a_j, b_i, c_w, d_v \) are the coefficients.

| No. of terms | No. of hidden layers | No. of considered M.M. | RMS\(^*\) | \(F_n/V\)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>400 ÷ 750</td>
<td>4</td>
<td>30</td>
<td>2.6 ÷ 3.6</td>
<td>0.6 ÷ 6.0</td>
</tr>
<tr>
<td>200 ÷ 250</td>
<td>1 to 4, mainly 3</td>
<td>20</td>
<td>3.7 ÷ 5.3</td>
<td>0.6 ÷ 6.0</td>
</tr>
<tr>
<td>150 ÷ 200</td>
<td>1 to 4, mainly 3</td>
<td>29</td>
<td>4.2 ÷ 5.7</td>
<td>0.6 ÷ 6.0</td>
</tr>
<tr>
<td>80 ÷ 150</td>
<td>1 to 3, mainly 2</td>
<td>21</td>
<td>4.6 ÷ 5.9</td>
<td>0.6 ÷ 6.0</td>
</tr>
<tr>
<td>116 3</td>
<td>Finally adopted M.M.</td>
<td>5.43</td>
<td>0.6 ÷ 6.0</td>
<td></td>
</tr>
</tbody>
</table>

\(^*\) RMS - according to aNETka. Manually calculated RMS is slightly different.

Note that no additional screening of obviously wrong data was done even after the derivation of the final model which clearly highlighted outliers. If that step was introduced and a new math model was then derived, the RMS would be considerably lower. Nevertheless, the quality of that model would be just about the same as of one used here. The RMS value is actually irrelevant, and is used only for the comparison of various models based on a same database.

26 models were derived for the wetted area coefficient (\( S/V^{2/3} \)), and the one finally selected has 23 polynomial terms with RMS of around 10 %.

\[
Y = \frac{\text{sig} \left( b_i + \sum_{j=1}^{2} B_j \cdot \text{sig} \left( a_j + \sum_{k=1}^{4} (A_{kj} \cdot (P_k \cdot X_k + R_k)) \right) \right) - G}{H}
\]

where:
- \( X_K = \{L/B, L/V^{1/3}, F_n/V, LCG/L\} \)
- \( Y = S/V^{2/3} \)
- \( G, H, P_k, R_k, A_{kj}, B_j, a_j, b_i \) are the coefficients.
A simple speed-dependent relation was required for the length of wetted area (L_w/L). There was no need to use ANN, it was derived simply by the application of the regression analysis. The finally selected mathematical model has 16 polynomial terms with RMS of around 9%.

\[ \frac{L_K}{L} = A \cdot F_{nV}^3 + B \cdot F_{nV}^2 + C \cdot F_{nV} + D \]

For all three equations, polynomial coefficients are given in Appendix 2. All three equations are approximately valid for the following range:

\[ 3.9 \leq \frac{L}{V^{1/3}} \leq 6.9 \]
\[ 2.5 \leq \frac{L}{B} \leq 4.7 \]
\[ 0.6 \leq F_{nV} \leq 3.5 \]
\[ 0.27 \leq \frac{LCG}{L} \leq 0.41 \]
\[ 12 \leq \beta \leq 24 \]

The boundaries of applicability of the models are depicted in Fig. 17, while the boundaries suitable for programming are given in Appendix 2. The boundaries are actually the surfaces which bound the multidimensional space.

The boundaries of applicability (Fig. 17) require additional discussion. Note:

a) TUNS and USCG Zones are formed according to hull-form and loading parameters of each particular model (dots) that form the series. Shown is Overlapping Zone as well.

b) Unreliable Zone (seen on the \( \beta = f(LCG/L) \) diagram only) is the zone where the mathematical model gives good results (the test data are well represented by the mathematical model) but the data is not logical, i.e. is most probably erroneous.

c) Rejected Zones (seen on the \( \beta = f(LCG/L) \) and \( \frac{L}{B} = f(LCG/L) \) diagrams) are the zones where the mathematical model is unstable (gives relatively bad results with saddles and humps for the interpolated values).

d) Lines denote the boundaries of applicability (borders) of the mathematical model (adjacent numbers indicate simple linear equations given in Appendix 2).

**Discussion concerning complex mathematical model**

The number of terms (116) in the polynomial seems large, but it should be noted that the Froude number is one
of independent variables. In comparable speed-independent mathematical models, for instance Radojcic (1991), $R/\Delta$ is evaluated for each Froude number separately, so for $F_nV$, range of up to 3.5 around 120 terms were needed (around 15 for each equation). In Radojcic et al (1997) speed-independent and speed-dependant math models were derived, albeit for semi-displacement NPL Series, both having around 150 polynomial terms. A disadvantage of speed-independent models, although often more accurate for one speed, is that the resistance computed at one speed is not directly linked to that at another speed. So to obtain $R/\Delta = f(F_nV)$ curve, smoothing is often necessary.

Note that both, Simple and Complex mathematical models for $L_K/L$, have the same number of polynomial terms and that the input variables – besides $F_nV$ – are $L/V^{1/3}$ and $LCG/L$ respectively, i.e. both model types depend on only two variables. Nevertheless, the second one is more accurate (RMS is 9 % vs. 16 %), confirming that $LCG/L$ is more influential parameter for $L_K/L$ evaluation than $L/V^{1/3}$.

More than a thousand 2D and 3D diagrams were constructed in order to check the quality of the derived mathematical model and the suggested boundaries of applicability. This facilitated examination of the 6-dimensional $R/\Delta$ surface from different angles. Some of typical diagrams deserve discussion. For instance, the diagrams depicted in Figs. 18 to 21 illustrate the correlation between the mathematical model and real experiments on the boundaries of applicability, where the match should be lower than in the middle of the applicability zone.

Discrepancies are obviously small. Moreover, it is almost certain that the mathematical model smoothed out some erroneous measurements, as for instance those in Fig. 19. In the same diagram, for instance, the discrepancies (measured to modelled) for $F_nV$ values of 0.6, 1.6 and 3.4 are 23 %, 12 % and 4 % respectively.

Note that the reliability of some TUNS data is questionable. In some cases there were just a few erroneous points which could easily be disregarded. But in other cases there were complete measurement sets that were obviously erroneous and not logical; see for instance Fig. 22 (curve for $\beta = 18$ is below $\beta = 12$, which is impossible). Often the mathematical model followed those erroneous data-points, see Fig. 23 which is typical for the so called Unreliable Zone.
The reasons for the appearance of the Rejected Zone are typically depicted in the Fig. 24 where the saddle was avoided by reducing part of the applicability zone. Namely, the mathematical model unsatisfactorily followed the experimental data.

Figs. 25 and 26 depict the discrepancies in the middle of the applicability zone. As with the previous diagrams (Figs. 18-21 and 23), the agreement between the TUNS and USCG Series \( R/\Delta \) values and those obtained by the mathematical model is fairly good. The mathematical model produced results that made more logical sense than the measurements due its inherent smoothing capabilities.

The quality of the derived mathematical model within the boundaries of applicability is described by 3D diagrams in Figs. 27 to 34. The presented cases are chosen ad hoc, with intention to provide the evidence that there were no instabilities in the model. So,

- Figs. 27 and 28 describe the influence of \( L/V_{1/3} \) variation
- Figs. 29 and 30 describe the influence of \( LCG/L \) variation
- Figs. 31 and 32 describe the influence of \( \beta \) variation
- Figs. 33 and 34 describe the influence of \( L/B \) variation.

Note that the diagram in Fig. 34 looks a bit wavy, and hence needs additional examination as shown in Figs. 35 and 36. Namely, the 3D diagram shown in Fig. 35 has \( L/V_{1/3} = 5.2 \), which is close to \( L/V_{1/3} = 5.5 \) of the 3D diagram in Fig. 34 (all other parameters being the same). For \( L/V_{1/3} = 5.2 \), however, there are test data curves \( (R/\Delta = f(FnV) \) of USCG models 5628, 5629 and 5630 with \( L/B = 3.37, 4.17 \) and 4.66 respectively. These are drawn on the surface evaluated by the mathematical model (see Figure 35).

Obviously, the discrepancies between the test-data (lines) and the mathematical model (surface) are small. This is illustrated better in Fig. 36. So, the mathematical model describes the USCG test data very well. The model test-data for \( R/\Delta \) of TUNS Series, however, are slightly different (see Figure 5), and hence strongly influence the surface for \( L/B \) values below 3.37, resulting in a hollow for \( L/B \) above 2.5 and below 3.37. In other words, the merger between the TUNS and USCG Series is not seamless. This is unavoidable drawback when two series are merged to form a new series, actually a joint data-base.

Note that some future mathematical model may be more accurate and have slightly broader boundaries of applicability. But it cannot be expected that it would be much better, since mathematical models cannot be more accurate than the measurement data they are based on, and, as already stated, some of the baseline data is obviously erroneous. Ergo, derivation of better models requires better measured data. Note, however, that a few erroneous measurements do not compromise the validity of the entire TUNS Series.
Fig. 27. The influence of $L/V^{1.3}$ on $R/\Delta$

Fig. 28. The influence of $L/V^{1.3}$ on $R/\Delta$

Fig. 29. The influence of $LCG/L$ on $R/\Delta$

Fig. 30. The influence of $LCG/L$ on $R/\Delta$

Fig. 31. The influence of $\beta$ on $R/\Delta$

Fig. 32. The influence of $\beta$ on $R/\Delta$
VERIFICATION OF THE MATHEMATICAL MODELS

The goodness of fit of both Simple and Complex mathematical models will be demonstrated for two planing hull models:

- TMB Model No. 4876 (SNAME Small Craft Data Sheet No. 14)
- TUNS Model 3018 (loading cases which were excluded from the database).

TMB Model No. 4876 represents Ray Hunt’s deep-V design of 52 ft LCSR (Landing Craft Swimmer Reconnaissance) and has chine and spray strips, as shown in Fig. 37. Calm-water test-data for resistance is given for a model of 3.25 ft, weighting 11.26 lb (see Figure 38). This figure also shows $R/\Delta$ evaluated using the Simple and Complex math models (evaluated for LCSR input parameters and scaled down to the same test conditions). Discrepancies are relatively small, and are smaller for the Simple model than for the Complex one, which is unexpected. The discrepancies, however, may be explained: LCSR’s spray strips separate the flow, and hence reduce the high speed resistance of deep-V hulls, whereas the math model does not include the effects of bottom spray strips.

For TUNS model No. 3018 ($L/B = 3$ and $\beta = 18$, Figure 3) two loading cases were evaluated, both for $L/V^{1/3} = 5.146$, but for $LCG/L = 0.329$ and 0.274, Figs. 39 and 40 respectively. The same figures also show the evaluated values for $R/\Delta$ (for model size of $L_{OA} = 0.69$ m and $\Delta = 1.825$ kg). Note that the
simple math model gave the same results for both positions of LCG, as is \( f(L/V_{1/3}) \) only. The Complex model, however, clearly depicts the resistance hump (Figure 40) due to LCG position closer to the transom, which is not the case with the Simple model. This also shows the advantage of the Complex math model compared to the Simple one (which is supposed to give relatively good results only if the input parameters are within the average values). Obviously, these two math models are not always equally representative with the experimental data.

Both mathematical models were examined for several test cases of models belonging to the systematic series and having arbitrary planing hull forms. Verification raised new questions and doubts regarding the quality of the benchmarks. Namely, test cases are often unreliable as are based on small models, see for instance Moore and Hawkins 1969 and Tanaka et al 1991. This subject is beyond the scope of this paper but merits future research.

CONCLUDING REMARKS

For evaluation of calm-water resistance \((R/\Delta)\) for standard displacement of \(\Delta = 100000\) lb two speed dependent mathematical models, Simple and Complex, were derived. Mathematical models for evaluation of the wetted area \((S/V^{2/3})\) and the length of wetted area \((Lk/L)\) were also derived (since they are needed for resistance evaluations when \(\Delta \neq 100000\) lb). In addition to the volume Froude number \((F_{v0})\), it is the slenderness ratio which is only used as the independent variable in the Simple model. In the Complex model, the slenderness ratio \((L/V_{1.3})\), the length to beam ratio \((L/B)\), the longitudinal centre of gravity \((LCG/L)\) and the deadrise angle \((\beta)\) are used. The Complex mathematical model is hence sensitive to variations of all mentioned independent variables. This is obviously not the case with the Simple model.

The Simple model can be used in the concept design phases, when it is practical and desirable to know the relationship between vessel’s length and weight, since other hull form parameters are usually unknown. The Complex model can be (and should be) used with other available planing-hull resistance evaluation models, such as for example, Radojcic (1985), Keuning et al (1993) and Savitsky (1964, 2006 and 2012). The proposed model covers hump speeds corresponding to \(F_{v0} \approx 0.6\) to 3.5. For higher planing speeds (of above \(F_{v0} \approx 3\)) the Savitsky model is recommended, unless the curved bow shape becomes wetted at low dynamic trim angles.

The derived math models are based on the well known USCG series, and almost unknown prismatic hull form of TUNS Series, both with wide transom which match the contemporary planing hull forms. Keuning (1993) and Radojcic (1985) resistance predictions are based on narrow transom Series 62 (having \(\beta = 12.5\) to 30 degrees), and wide transom Series 65-B and narrow transom Series 62 (\(\beta = 12.5\) to 25 degrees), respectively. The here derived Complex model may be incorporated in other power evaluation routines (numerical towing tank performance predictions) or optimization routines with the aim of obtaining the best dimensions, as for example given in Radojcic (1991). It should be noted that tank testing is rarely used in design of small craft due to the cost of tests relative to the cost of the vessel, hence various numerical towing tank performance predictions are used for all design phases.

Furthermore, it seems that the regression analysis is more convenient than artificial neural network (ANN) for simpler relations/equations. For complex relations with many polynomial terms, the regression analysis requires more time and higher levels of skill which is not so pronounced with ANN. Moreover, the step-by-step procedure and regression analysis allow screening of less significant polynomial terms (resulting with smaller polynomial equation) which is not the case with ANN where the number of terms is defined at the very beginning.
The most significant disadvantage of the derived math models is that, in addition to the USCG Series, they are based on the TUNS Series—which is based on relatively small models—hence for some cases the database used is not reliable. It is believed, however, that both math models have smoothed out the inconsistencies and erroneous measurements. So, in some cases mathematical models might be more reliable than the original data.

Whereas some interesting behaviours were observed when checking the quality of the mathematical model and comparing the evaluated values with the model test data of arbitrary hull forms, it was decided to leave that analysis for future work.

Acknowledgement

Authors gratefully acknowledge help and support provided by the high speed and planing craft guru, Mr. Donald Blount who unselfishly shared his knowledge and 50 years experience. Concerning this paper, he was involved in surfacing representative hull form and loading parameters, the subject which is much more delicate than it seems, as well as in giving several useful comments which improved paper’s quality.

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CONTACT WITH THE AUTHORS

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Dept. of Naval Architecture,
Belgrade, SERBIA
2) NAVAR,
Herceg Novi, MONTENEGRO
APPENDIX 1 – Simple Mathematical Model

\[
\frac{R}{\Delta} = A \cdot F_{nV}^3 + B \cdot F_{nV}^2 + C \cdot F_{nV} + D
\]

\[
A = -0.0048694 \cdot \left( \frac{L}{V^{1/3}} \right)^4 + 0.1057838 \cdot \left( \frac{L}{V^{1/3}} \right)^3 - 0.8432151 \cdot \left( \frac{L}{V^{1/3}} \right)^2 + 2.8994541 \cdot \left( \frac{L}{V^{1/3}} \right) - 3.5683179
\]

\[
B = 0.0301221 \cdot \left( \frac{L}{V^{1/3}} \right)^4 - 0.6562651 \cdot \left( \frac{L}{V^{1/3}} \right)^3 + 5.2443776 \cdot \left( \frac{L}{V^{1/3}} \right)^2 - 18.0560600 \cdot \left( \frac{L}{V^{1/3}} \right) + 22.1656778
\]

\[
C = -0.0508810 \cdot \left( \frac{L}{V^{1/3}} \right)^4 + 1.1119496 \cdot \left( \frac{L}{V^{1/3}} \right)^3 - 8.9006694 \cdot \left( \frac{L}{V^{1/3}} \right)^2 + 30.6066779 \cdot \left( \frac{L}{V^{1/3}} \right) - 37.2112557
\]

\[
D = 0.0209140 \cdot \left( \frac{L}{V^{1/3}} \right)^4 - 0.4573261 \cdot \left( \frac{L}{V^{1/3}} \right)^3 + 3.6580101 \cdot \left( \frac{L}{V^{1/3}} \right)^2 - 12.5431467 \cdot \left( \frac{L}{V^{1/3}} \right) + 15.14353
\]

\[
\frac{S}{V^{2/3}} = A \cdot F_{nV}^3 + B \cdot F_{nV}^2 + C \cdot F_{nV} + D
\]

\[
A = 0.0197989 \cdot \left( \frac{L}{V^{1/3}} \right)^3 - 0.2876721 \cdot \left( \frac{L}{V^{1/3}} \right)^2 + 1.3944044 \cdot \left( \frac{L}{V^{1/3}} \right) - 2.1175915
\]

\[
B = -0.1612880 \cdot \left( \frac{L}{V^{1/3}} \right)^3 + 2.3295698 \cdot \left( \frac{L}{V^{1/3}} \right)^2 - 11.3511782 \cdot \left( \frac{L}{V^{1/3}} \right) + 18.0264798
\]

\[
C = 0.4226973 \cdot \left( \frac{L}{V^{1/3}} \right)^3 - 6.1030996 \cdot \left( \frac{L}{V^{1/3}} \right)^2 + 29.8350864 \cdot \left( \frac{L}{V^{1/3}} \right) - 49.7163001
\]

\[
D = -0.4432887 \cdot \left( \frac{L}{V^{1/3}} \right)^3 + 6.7794188 \cdot \left( \frac{L}{V^{1/3}} \right)^2 - 33.1423777 \cdot \left( \frac{L}{V^{1/3}} \right) + 58.8177152
\]

\[
\frac{L_k}{L} = A \cdot F_{nV}^3 + B \cdot F_{nV}^2 + C \cdot F_{nV} + D
\]

\[
A = 0.0010024 \cdot \left( \frac{L}{V^{1/3}} \right)^3 - 0.0165489 \cdot \left( \frac{L}{V^{1/3}} \right)^2 + 0.0910387 \cdot \left( \frac{L}{V^{1/3}} \right) - 0.1630761
\]

\[
B = -0.0021325 \cdot \left( \frac{L}{V^{1/3}} \right)^3 + 0.0437025 \cdot \left( \frac{L}{V^{1/3}} \right)^2 - 0.3171667 \cdot \left( \frac{L}{V^{1/3}} \right) + 0.7839794
\]

\[
C = -0.0049667 \cdot \left( \frac{L}{V^{1/3}} \right)^3 + 0.0482171 \cdot \left( \frac{L}{V^{1/3}} \right)^2 + 0.0626019 \cdot \left( \frac{L}{V^{1/3}} \right) - 1.1483516
\]

\[
D = -0.0086702 \cdot \left( \frac{L}{V^{1/3}} \right)^3 + 0.1543888 \cdot \left( \frac{L}{V^{1/3}} \right)^2 - 0.9891214 \cdot \left( \frac{L}{V^{1/3}} \right) + 3.2719934
\]
APPENDIX 2 – Complex Mathematical Model

Boundaries of applicability suitable for programming

<table>
<thead>
<tr>
<th>No.</th>
<th>0.27 ≤ LCG/L &lt; 0.33</th>
<th>L/V^{1/3} ≥ -6.66667 · LCG/L+ 6.1</th>
<th>No.</th>
<th>0.39 ≤ LCG/L ≤ 0.41</th>
<th>L/B ≥ 45 · LCG/L - 15.05</th>
</tr>
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<tr>
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<td>0.39 ≤ LCG/L ≤ 0.41</td>
<td>L/V^{1/3} ≥ 65 · LCG/L- 21.45</td>
<td>11</td>
<td>0.27 ≤ LCG/L &lt; 0.36</td>
<td>L/B≤ 3.5</td>
</tr>
<tr>
<td>3</td>
<td>0.27 ≤ LCG/L &lt; 0.33</td>
<td>L/V^{1/3} ≥ 6.66667 · LCG/L+ 4.7</td>
<td>12</td>
<td>3.5 &lt; L/B ≤ 4.7</td>
<td>LCG/L ≥ 0.36</td>
</tr>
<tr>
<td>4</td>
<td>0.39 ≤ LCG/L ≤ 0.41</td>
<td>L/V^{1/3} ≤ -35 · LCG/L+ 20.55</td>
<td>13</td>
<td>0.35 ≤ LCG/L ≤ 0.41</td>
<td>β ≥ 100 · LCG/L - 23</td>
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<tr>
<td>5</td>
<td>2.5 ≤ L/B &lt; 4.0</td>
<td>L/V^{1/3} ≥ 0.866667 · L/B+ 1.73333</td>
<td>14</td>
<td>0.39 ≤ LCG/L ≤ 0.41</td>
<td>β ≤ -150 · LCG/L + 82.5</td>
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<td>L/V^{1/3} ≥ 5.2</td>
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<td>12° ≤ β ≤ 14°</td>
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<td>2.5 ≤ L/B ≤ 3.5</td>
<td>L/V^{1/3} ≤ 1.3 · L/B+ 2.35</td>
<td>16</td>
<td>14° &lt; β ≤ 18°</td>
<td>L/B≤ 0.3 · β - 0.7</td>
</tr>
<tr>
<td>8</td>
<td>3.5 &lt; L/B ≤ 4.7</td>
<td>L/V^{1/3} &lt; -0.583333 · L/B+ 8.94167</td>
<td>17</td>
<td>21° ≤ β ≤ 24°</td>
<td>L/B≤ -0.4 · β + 13.1</td>
</tr>
<tr>
<td>9</td>
<td>0.27 ≤ LCG/L ≤ 0.35</td>
<td>L/B ≥ -5 · LCG/L + 4.25</td>
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Resistance to weight ratio – R/∆

\[
Y = \frac{\text{sig}\left(\sum_{i=1}^{3} B_i \times \text{sig}\left(\sum_{j=1}^{5} C_{ij} \times \text{sig}\left(\sum_{k=1}^{5} A_{ijk} \times (P_k X_k + R_k)\right)\right)\right)}{\text{H}} - G
\]

where:

\[X_k = \{L/B, L/V^{1/3}, F_{nv}, LCG/L, \beta\}\]

G, H, P, R, A_{ij}, B_{ij}, C_{ij}, D_{ij} are polynomial coefficients that follow:

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<td>G</td>
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Wetted area coefficient – $S/V^{2/3}$

$$Y = \frac{\text{sig} \left( b_1 + \sum_{j=1}^{2} B_j \cdot \text{sig} \left( a_j + \sum_{k=1}^{4} (A_{kj} \cdot (P_k \cdot X_k + R_k)) \right) \right) - G}{H}$$

where:

$$X_k = \{ L/B, F_{V}, L/V^{1/3}, LCG/L \}$$

$$Y = R/V^{2/3}$$

$G$, $H$, $P_k$, $R_k$, $A_{kj}$, $B_j$, $a_j$, $b_i$ are polynomial coefficients that follow:

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<th>$LCG/L$</th>
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<td>$L/K$</td>
<td>$A \cdot F_{V}^{3} + B \cdot F_{V}^{2} + C \cdot F_{V} + D$</td>
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</table>

<table>
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<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
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<tr>
<td>$A$</td>
<td>-19.861·$\left( \frac{LCG}{L} \right)^{3}$ + 17.895·$\left( \frac{LCG}{L} \right)^{2}$ - 5.0552·$\left( \frac{LCG}{L} \right)$ + 0.4443</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>198.98·$\left( \frac{LCG}{L} \right)^{3}$ - 188.93·$\left( \frac{LCG}{L} \right)^{2}$ + 57.04·$\left( \frac{LCG}{L} \right)$ - 5.4136</td>
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<tr>
<td>$C$</td>
<td>-548.66·$\left( \frac{LCG}{L} \right)^{3}$ + 541.85·$\left( \frac{LCG}{L} \right)^{2}$ - 171.02·$\left( \frac{LCG}{L} \right)$ + 16.936</td>
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<tr>
<td>$D$</td>
<td>347.6·$\left( \frac{LCG}{L} \right)^{3}$ - 359.66·$\left( \frac{LCG}{L} \right)^{2}$ + 120.79·$\left( \frac{LCG}{L} \right)$ - 12.097</td>
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A simplified method for calculating propeller thrust decrease for a ship sailing on a given shipping lane

Katarzyna Zelazny, Ph. D., West Pomeranian University of Technology Szczecin, Poland

ABSTRACT

During ship sailing on rough water, relative ship motions can be observed which make the propeller emerge from the water, and decrease its thrust as a consequence. The article presents a simplified method for calculating the thrust decrease and the time of propeller emergence from water for the ship on a regular or an irregular wave. The method can be used for predicting the operating speed of the ship on a given shipping lane.

Keywords: ship motions on rough water; propeller emergence; thrust decrease

INTRODUCTION

As a direct effect of ship sailing on rough water, ship's heaving, rolling and pitching continuously take place in response to waves being their source. These motions can provoke other dangerous phenomena, such as accelerations and relative motions, which act continuously, and those occurring occasionally, like deck flooding, slamming, or propeller emergence from water. The latter phenomena result from, among other sources, relative ship motions, and in those cases the object of analyses is the frequency of their appearance in one hour or per 100 waves. The propeller emergence is dangerous for the entire ship propulsion system, in particular it is the source of thrust decrease which finally results in speed decrease of the ship sailing on rough water (this effect can also be caused by other causes) [6]. Calculating the ship thrust and speed decrease requires not only the information about the frequency of the propeller emergence, per hour for instance, but also on the scale and duration times of this emergence on a given shipping lane. Based on these data the thrust decrease can be calculated, and then the ship speed decrease caused by the propeller emerging from water.

The problem of propeller thrust decrease during ship sailing on rough water has been the object of study in numerous publications, for instance in [3] – a study of the effect of waves, but without propeller emergence, on the speed of the wake current and thrust, [2] – a study of thrust decrease during propeller emergence, and [4] – where the thrust decrease caused by the bow thruster tunnel emerging from water was examined. In Refs. [7] and [8] their authors also analysed the approximate effect of propeller emergence on the ship speed decrease. However, these publications do not provide information on the current value of the propeller thrust when the ship sails in given weather (sea) conditions and when its course is in certain relation to the direction of waves.

For the ship sailing on a given shipping lane the propeller emergence and thrust decrease take place when the wave is sufficiently high. Relevant analyses have revealed [8] that this situation occurs occasionally and the propeller thrust decrease leads to the decrease of operating speed of the ship by less than ten percent (which obviously depends on parameters of sea waves and navigating characteristics of the ship).

The article presents a simplified method for calculating the thrust decrease caused by the propeller emergence from water on regular and irregular wave. The presented method can be applied for predicting the operating speed of the ship sailing on a given shipping lane.

RELATIVE SHIP MOTION AND PROPELLER EMERGENCE ON REGULAR WAVE

Pitching, rolling, and heaving of the ship sailing on rough water generate its relative motions. The absolute vertical ship dislocation resulting from these motions is:

\[ S_{zp}(t) = Z(t) + y_p \Phi(t) - x_p \Theta(t) \]  

while the relative dislocation is:

\[ R_{zp}(t) = S_{zp}(t) - \zeta(t) \]

where:

- \( Z(t), \Phi(t), \Theta(t) \) – ship heaving, rolling and pitching on regular wave,
$x_P, y_P$ — coordinates of point $P$ fixed to the ship, for which the relative vertical motion is calculated. In this case the point is situated at the propeller blade tip in its upper position, Fig. 2.

$\zeta(t)$ — ordinate of the regular sinusoidal wave which approaches the ship at the angle $\beta_w$ (Fig. 1).

The relative motion $R_{z_P}$ of the point $P$ (propeller blade tip in its upper position - Fig. 2) is calculated based on the linear theory of ship motions, hence Equation (2) can have the form:

$$R_{z_P}(t) = R_{z_P A} \sin[\omega_E t - \delta_{Rz_P}(\omega_E)]$$

(3)

where:

$R_{z_P A}$ — amplitude of the relative vertical motion of point $P$,

$\delta_{Rz_P}(\omega_E)$ — phase shift angle between the ordinate of the relative motion and that of the wave motion.

Based on the relative motion, Equation (2) or (3), and the propeller position (more precisely: propeller blade tip – point $P$, Fig. 2) we can calculate when, to which height and for how long the propeller will emerge from water. The propeller emergence to the height $h_{ws}$ will take place when:

$$h_{ws}(t) = R_{z_P}(t) - T_{zS}(t) > 0$$

(4)

where:

$T_{zS}$ — the draught of the propeller blade tip in its upper position:

$$T_{zS} = T_{P0} - 0.5D_p$$

(5)

$T_{P0}$ — propeller shaft draught (Fig. 2),

$D_p$ — propeller diameter.

A sample time-history of propeller emergence on regular wave, calculated for the ship having parameters given in Table 1, is shown in Fig. 3.

**Tab. 1. Technical parameters and dimensions of the ship**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars</td>
<td>$L_{pp} = 138.0$ m</td>
</tr>
<tr>
<td>Width</td>
<td>$B = 23.0$ m</td>
</tr>
<tr>
<td>Draught</td>
<td>$T = 8.5$ m</td>
</tr>
<tr>
<td>Displacement</td>
<td>$\nabla = 21 411$ m$^3$</td>
</tr>
<tr>
<td>Speed</td>
<td>$V = 14.24$ w</td>
</tr>
<tr>
<td>Propeller diameter</td>
<td>$D_p = 5.0$ m</td>
</tr>
<tr>
<td>Propeller revolutions</td>
<td>$n_p = 110$ rpm</td>
</tr>
<tr>
<td>Propeller shaft draught</td>
<td>$T_{P0} = 5.9$ m</td>
</tr>
<tr>
<td>Position of the propeller disc</td>
<td>$x_p = -68.16$ m</td>
</tr>
</tbody>
</table>

The time-history of the ordinate of the propeller blade tip emergence $h_{ws}(t)$ has made the basis for calculating its average value $h_{w,avg}$ (Fig. 3):

$$h_{(w)avg} = \frac{1}{T_{ws}} \int_{t_1}^{t_3} h_{ws}(t)dt$$

(6)

where:

$$h_{ws}(t) = \begin{cases} R_{z_P}(t) & for \ t \in (t_1, t_2) \\ 0 & for \ t \in (t_2, t_3) \end{cases}$$

(7)

$t_1, t_2$ — beginning and end of propeller blade tip emergence, Fig. 3,

$t_3$ — end of relative motion time period $R_{z_P}(t)$, Fig. 3.

The average propeller emergence was calculated for different amplitudes $\zeta_A$ and frequencies $\omega$ of the regular wave, different directions of wave approach with respect to the ship, and different assumed ship speeds. Fig. 4 shows one of the analysed cases. The differences of the average propeller emergence values (Fig. 4) which are observed for different
wave frequencies $\omega$ result from amplitude characteristics of ship motions on regular wave (Table 1) for which the calculations were performed.

The rotational speed of the analysed ship is $n = 110$ [rpm] (Table 1). The motion of the propeller tip during the emergence process taking place at this speed is shown in Fig. 3. Based on the time-history of position changes of the propeller blade tip we can conclude that when emerging from water, the propeller can accomplish about twenty full revolutions. For a given emergence height $h_{\text{ws}}$ the surface of the emerging propeller during its rotations undergoes small changes resulting from the shape of the propeller outline. In order to simplify the problem it was assumed that for about ten revolutions the average surface of the propeller part emerging from water is proportional to the average emergence height $h_{\text{ws}}$ (6), Fig. 3.

The surface of the propeller blades is defined using the expanded blade area ratio:

$$a_{E} = \frac{A_{E}}{A_{0}}$$  \hspace{1cm} (8)

where:

$A_{E}$ – expanded area of propeller blades,

$A_{0}$ – propeller disc,

while for the emerging propeller the coefficient $a_{E}^{*}$ is defined in the form:

$$a_{E}^{*} = 1 - \left(\frac{A_{E}}{A_{0}}\right)$$  \hspace{1cm} (9)

which depends on the ratio $(h_{\text{ws}}/D_{p})$. This coefficient was calculated using the emerged propeller surface $A_{E_{0}}$ (Fig. 3).

![Fig. 3. Relative motion, emergence height, and the time-history of position changes of the propeller blade tip in its upper position](image)

The effect of the propeller emergence process on the area of its emerged segment is shown in Fig. 5.

![Fig. 4. Average propeller emergence height on regular wave for different wave amplitudes $\zeta_{w}$ and directions $\beta_{w}$ with respect to the ship ($V = 6$ m/s)](image)

**THE EFFECT OF PROPELLER EMERGENCE ON REGULAR WAVE ON PROPELLER THRUST**

The hydrodynamic characteristics of the propeller having basic parameters given in Table 1 and totally submersed in water are shown in Fig. 6.

![Fig. 5. Emerged propeller segment area vs. emergence height](image)

The thrust of the propeller partially emerged from water was calculated using the vortex surface theory based algorithm [5] which can be used for calculating the thrust on a propeller blade segment in different angular positions of the segment. This approach provides opportunity to calculate the thrust distribution on the propeller blade (Fig. 7). The calculations making use of the above algorithm were performed based on the following simplifying assumptions:

- the thrust decrease results from reducing to zero the thrust distribution over the blade surface part emerging from water,
- neglected is the effect of the division surface between the water and the air on the propeller blade,
neglected is the water density change resulting from its saturation with air during entering of the propeller blade into water and leaving it.

Area changes of the propeller blade segment emerged from the water for different emergence heights and angular positions are shown in Fig. 8.
The time-history of the area changes of the emerging propeller blade segment is shown in Fig. 9. For a given level of propeller emergence the thrust will change in the same way. The average total thrust values for different propeller emergence values are shown in Fig. 10, while Fig. 11 shows the effect of the parameters and ship-related direction of the regular wave, being the cause of propeller emergence, on the thrust.

**PROPELLER EMERGENCE ON IRREGULAR WAVE**

When calculating the relative ship motion on irregular wave we have to take into account the phase shift between the ordinate $\xi(t)$ of the irregular wave and that of the irregular ship motion $u_i(t)$, as well as the phase shifts between particular forms of ship motions composing its relative motion. The theory of pulse transmission function can be used for this purpose [9].

With the aid of the pulse transmission function the potential damping $[b_{k,l}(t)]$ of ship motions on regular wave can be written as:

$$b_{(k,l)}(t)[u_{(l)}(t)] = \int_0^\infty [K_{(k,l)}(\tau)]u_{(l)}(t-\tau)\,d\tau$$  \hfill (10)

and the hydrodynamic masses $[m_{k,l}(t)]$ have the following form:

$$[m_{(k,l)}(t)] = [m_{(k,l)}(\tilde{\omega})] + \frac{1}{\tilde{\omega}} \int_0^\infty [K_{(k,l)}(t)\sin(\tilde{\omega}t)]\,dt$$  \hfill (11)

where:

$[K_{(k,l)}]$ – matrix of pulse transmission functions,

$[m_{(k,l)}(\tilde{\omega})]$ – matrix of generalised hydrodynamic masses from the regular wave with frequency $\tilde{\omega}$,

$\tilde{\omega}$ – arbitrarily selected frequency of the regular wave,

$\tau$ – time interval.

The form of the function $K_{(k,l)}(t)$ for the watercraft has been given in [11] as:

$$K_{(k,l)}(t) = \frac{2}{\pi} \int_0^\infty [b_{(k,l)}(\omega)]\cos(\omega t)\,d\omega$$  \hfill (12)

where $[b_{(k,l)}(\omega)]$ is the matrix of generalised coefficients of potential ship motion damping on regular wave.

Taking into account Equation (10), the ship motions on irregular wave can be described by the following equation system:

$$([M_{(k,l)}] + [m_{(k,l)}(t)])\{u_{(l)}(t)\} + \int_0^\infty [K_{(k,l)}(\tau)]\{u_{(l)}(t-\tau)\}\,d\tau + [C_{H(k,l)}(t)]\{u_{(l)}(t)\} = \{F_{(k,l)}(t)\}$$  \hfill (13)

where:

$[M_{k,l}(t)]$ and $[K_{(k,l)}(\tau)]$ are given by Eqs. (11) and (12), respectively,

$[C_{H(k,l)}]$ – matrix of generalised hydrostatic coefficients of restoring forces,

$\{u_{(l)}\}$ – vector of generalised accelerations generated by ship motions on irregular wave,

$\{\dot{u}_{(l)}\}$ – vector of generalised velocities generated by ship motions on irregular wave,

$\{\ddot{u}_{(l)}\}$ – vector of generalised dislocations generated by ship motions on irregular wave,

$\{F_{(k,l)}(t)\}$ – vector of generalised wave excitation forces generated by ship motions on irregular wave.

A solution method for Equations (13) and calculation of ship motions (and relative motions) on irregular wave are given in Ref. [12].

**SAMPLE CALCULATIONS OF PROPELLER EMERGENCE ON IRREGULAR WAVE**

The relative ship motion calculations were performed for the irregular wave defined by the wave energy spectral density function acc. to ITTC:

$$S_{\xi\xi}(\omega) = A \cdot \omega^{-5} \cdot \exp(-\frac{B}{\omega^4})$$  \hfill (14)

where:

$A = 173 \frac{H_s^2}{T_i}$, $B = 691 \frac{1}{T_i}$, $H_s$ – meaningful wave height,

$T_1$ – average characteristic time period.

The relative vertical motion on irregular wave is the following:

$$R_z(t) = S_{\xi}(t) - \xi(t)$$  \hfill (15)

where:

$\xi(t)$ – ordinate of the irregular wave,

$S_{\xi}(t)$ – ordinate of the absolute vertical motion:

$$S_{\xi}(t) = Z(t) + y_p\phi(t) - x_p\theta(t)$$  \hfill (16)

while $Z(t)$, $\phi(t)$, $\theta(t)$ are the ordinates of ship heaving, rolling and pitching on irregular wave for time $t$.

The results of computer simulations of the time-histories of the random wave ordinate, and the absolute and relative ordinates of the motion of the propeller blade tip, as well as the time-history of the propeller blade tip emergence process on irregular wave are shown in Figs. 12 ÷ 15.

The time-history of the propeller blade tip emergence height $h_{(k,l)}(t)$ (Fig. 15) has made the basis for calculating its average value $h_{(k,l)}$ for the time interval equal to 3 hours ($T = 10800$ s):
Fig. 12. Time-history of the random ordinate of the irregular wave for 8°B (\(H_s = 5.25\) m, \(T_1 = 8.5\) s) and \(\beta_w = 0^\circ\)

Fig. 13. Time-history of the ordinate of the absolute propeller blade tip motion on irregular wave for 8°B (\(H_s = 5.25\) m, \(T_1 = 8.5\) s) and \(\beta_w = 0^\circ\)

Fig. 14. Relative vertical motion of the propeller blade tip on irregular wave for 8°B (\(H_s = 5.25\) m, \(T_1 = 8.5\) s) and \(\beta_w = 0^\circ\)

Fig. 15. Propeller blade tip emergence on irregular wave for 8°B (\(H_s = 5.25\) m, \(T_1 = 8.5\) s) and \(\beta_w = 0^\circ\); \(h_{\text{w(\text{cn})}} = 0.03\) m

Fig. 16. Time-history of propeller thrust changes on irregular wave for 8°B (\(H_s = 5.25\) m, \(T_1 = 8.5\) s) and \(\beta_w = 0^\circ\)
where:

\[ h_{(ws)Ir} = \frac{1}{T} \int_0^T h_{ws}(t) \, dt \]  

\[ h_{ws}(t) = \begin{cases} R_{wp}(t) & \text{for } R_{wp}(t) \geq 0 \\ \text{otherwise} & \end{cases} \]

Like in case of the regular wave, the time of propeller emergence on irregular wave (Fig. 15) is sufficiently long for the propeller to accomplish a number of full revolutions (Fig. 3). The instantaneous propeller thrust decrease on irregular wave was also calculated in the way similar to that used for the regular wave (Fig. 16).

The average propeller thrust decrease on irregular wave was calculated for different sea conditions and different wave directions with respect to the ship, and for the assumed ship speeds. Figure 17 shows sample results obtained for one of the analysed cases.

![Fig. 17. Thrust changes during propeller emergence on irregular wave for different sea conditions and wave directions \( \beta \), with respect to the ship \((V = 6 \text{ m/s})\)](image)

The obtained results referring to the propeller thrust decrease during ship motions (Table 1) have been compared with those presented in Ref. [10], Fig. 18. That publication presents only the propeller thrust decrease as a function of propeller emergence, without any information on parameters of the ship and the propeller. In this context an opinion seems to be reasonable that the results given in Ref. [10] have been averaged for different ships and propellers. Despite the adopted simplifying assumptions, the here presented results reveal sufficient accuracy in propeller thrust decrease evaluation for known geometric parameters of both the ship and the propeller.

### CONCLUSIONS

- During preliminary ship design, a crucial task is to predict accurately the operational ship speed on a given shipping lane. This speed is affected by, among other factors, the propeller thrust decrease resulting from propeller emergence during ship motions on rough water.
- The article presents a simplified method for evaluating propeller thrust decrease during ship motions on regular and irregular wave. This method makes it possible to calculate the propeller thrust decrease for an arbitrary ship and propeller, and for given wave parameters occurring on the shipping lane.
- Despite certain simplifications, the presented method gives satisfactory results (they were compared with relevant results available in the literature), the accuracy of which is sufficiently high to use it for predicting the operating speed of the ship on an arbitrary shipping lane at the preliminary design stage.

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A numerical model to simulate the motion of a lifesaving module during its launching from the ship’s stern ramp

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ABSTRACT

The article presents a numerical model of object motion in six degrees of freedom (DoF) which is intended to be used to simulate 3D motion of a lifesaving module during its launching from a ship using a stern ramp in rough sea. The model, of relatively high complexity, takes into account both the motion of the ship on water in changing sea conditions, and the relative motion of the ramp with respect to the ship. The motion of the ramp changes and strongly depends on its constructional and geometrical parameters. The presented model takes into account the displacement of the submerged part of the ramp, as well as its damping in the water and the interaction with the module moving on it. The results of test simulation of a module launching from the ship in still water are included.

Keywords: lifeboat; rescue boat launch; innovation; launch ramp aft; computational model for simulation

INTRODUCTION

Protection and saving of human life is one of basic issues which have to be solved by designers of marine watercraft, especially passenger ships.

Lifesaving systems used for evacuating people, especially from large passenger ships, have evolved starting from relatively simple open lifeboats and life rafts, up to presently used unsinkable closed lifeboats revealing high strength and fire resistance which are extended overboard and launched at a controlled speed using board davits. These davits provide opportunity to extend the boat overboard and launch it even in the lack of power supply, and at ship’s heel reaching 20° and trim up to 10° using gravitational forces or the collected energy.

An important aspect to be taken in to account when developing these systems is the tendency to shorten the evacuation time and, at the same time, to secure safe and relatively comfortable getting people into the lifesaving units.

This problem was part of the European project entitled SAFECRAFTS pt. “Safe Abandoning of Ships - improvement of current Life Saving Appliances Systems”, which was carried out in years 2004-2009 and the participant of which was the Faculty of Ocean Engineering and Ship Technology, Gdansk University of Technology.

Within the framework of this project a number of concepts of novel LSA systems were developed, as described in [1, 2, 3] and [4]. In two of those systems which look most interesting an open stern ramp/door was applied, over which the launched lifesaving boats or other watercraft units with people inside are to slide into water in the direction opposite to the motion of the endangered ship.

One of these systems, shown in Fig. 1 and described in detail in [3], provides opportunities for fast and safe evacuation of people using only gravitational forces. The rescued people get simultaneously into all lifeboats situated on different decks, after which a system is started to lower the boats at a controlled speed. When the boats reach the level of the ship’s slip, they are hooked in, one by one, and freely slide down the ramp to the water.

In the second system, developed by Fassmer and schematically shown in Fig. 2, the watercraft units are situated in rows on the lower deck. People get into all boats at the same time, and then using the already accumulated energy, or that delivered from an additional electric power generator, the boats are rolled to the ramp from which they slide down to the water. The ramp has a three-segment structure, but when the boats with people slide down to water all three segments are blocked in a one-plane position. The last ramp segment has an air tank connected to its bottom side, to allow the ramp to adapt its inclination angle to the water level overboard, which changes mainly to ship rolling or pitching motions and sea waving. The largest ramp inclination angle is 36°.

In these two systems the most dynamic and dangerous evacuation stage is sliding the unit with people down the ramp in rough sea. It is worth stressing, however, that modelling this evacuation stage in the second system is more difficult and general, and that is why it was selected for realisation as one of separate research tasks in the abovementioned project.
The article presents the results of the task performed by the authors which concerned developing a comprehensive mathematical model to simulate the motion of the lifesaving module during its launching from the ship in rough sea. Moreover, the results of test calculations of sample launching from the ship in still water are included.
DESCRIPTION OF THE MATHEMATICAL MODEL

In the numerical model developed by the authors, the moving basic system components, i.e. the lifesaving module and the moving part of the ramp, are generally modelled as rigid bodies with six degrees of freedom which are subject to: the action of the environment (water, wind, etc.), the reactions of neighbouring objects (collisions), and the reactions coming from motion constraints, such as the hinges which fix the moving part of the ramp, for instance.

Moreover, certain elements (nodes) are identified in the lifesaving module which are used for detecting collisions with other objects (mainly the ramp). The interaction is of the elastic-plastic nature with damping. Node deformations are not permanent – at the next collision their positions with respect to the local coordinate system of the module remain unchanged.

Equations of motion of the rigid body (lifesaving module)

The motion of a rigid body in six degrees of freedom is described by the following equations:

\[
\begin{align*}
\frac{d}{dt}(MV_x) &= F_x \\
\frac{d}{dt}(MV_y) &= F_y \\
\frac{d}{dt}(MV_z) &= F_z \\
I_{xx0} \frac{d}{dt}\omega_x &= (I_{yy0} - I_{zz0})\omega_y \omega_z = M_x \\
I_{yy0} \frac{d}{dt}\omega_y &= (I_{zz0} - I_{xx0})\omega_z \omega_x = M_y \\
I_{zz0} \frac{d}{dt}\omega_z &= (I_{xx0} - I_{yy0})\omega_x \omega_y = M_z
\end{align*}
\]  

where:
- \(M\) – mass of the rigid body,
- \(V_x, V_y, V_z\) – velocity vector components,
- \(\omega_x, \omega_y, \omega_z\) – angular velocity components,
- \(I_{xx0}, I_{yy0}, I_{zz0}\) – moment of inertia with respect to \(x_0, y_0, z_0\) axis, respectively
- \(F_x, F_y, F_z\) – force vector components,
- \(M_x, M_y, M_z\) – moment vector components,
- \(x, y, z\) – coordinates of the “absolute” coordinate system fixed to the rigid body.
- \(x_0, y_0, z_0\) – coordinates of the local coordinate system

The forces and moments are calculated as the sum of the reactions of interactions with other objects, the hydrodynamic reactions, and the gravitational forces.

Hydrostatic and hydrodynamic forces

The hydrostatic forces were calculated as the integrals of the hydrostatic pressure function over the hull surface. The hull of the module and the pontoon plating were modelled using tetragonal panels. In order to calculate the hydrodynamic forces acting on partially submerged panels, the submerged part and the centre of the panel were to be calculated. The hydrostatic force acting on a single panel was calculated from the formula:

\[
R_{HS} = -\rho_w g h_s S_{wet} n
\]

where:
- \(R_{HS}\) – hydrostatic force vector,
- \(\rho_w\) – water density,
- \(g\) – acceleration of gravity
- \(h_s\) – submersion of the centre of the wetted panel surface
- \(S_{wet}\) – area of the wetted panel part
- \(n\) – unit normal vector

The hydrodynamic reaction is the sum of friction and pressure forces. The total reaction is the sum of the hydrodynamic forces acting on all panels. For a single panel the hydrodynamic force is calculated using the formula:

\[
R_{HD} = -\frac{1}{2} \rho_w (C_F V_T + C_P V_n)V_n S_{wet}
\]

where:
- \(R_{HD}\) – hydrodynamic resistance,
- \(C_F, C_P\) – friction and pressure force coefficients,
- \(V_T, V_n, V_n\) – tangent and normal velocity vectors and absolute values, respectively.

Definitions of \(V_T\) and \(V_n\) are given as:

\[
V_n = (V \cdot n)n; V_T = V - V_n
\]

Hydrostatic and hydrodynamic forces

On the hull of the lifesaving module, a number of bearing nodes were selected which were the objects of action of the ramp pressure forces. The value of the force is defined by the linear function of the crossing distance of the basic (undeformed) ramp surface by the bearing node, see Fig. 3 and Fig. 4. This linear function has been limited by the maximal pressure reaction, introduced to limit modelling of the bearing pressure forces to the range observed in real conditions (for instance due to the loss of stability of the structure, or exceeding the yield point).

The pressure force acting on the ramp:

\[
R_N = \begin{cases} 
E_{spr} \Delta s \mathbf{n} & \Delta s \leq s_{spr} \\
R_{MAX} \mathbf{n} & \Delta s > s_{spr}
\end{cases}
\]

where:
- \(E_{spr}\) – modulus of elasticity [N/m],
- \(\Delta s\) – distance between the bearing node and the base surface, see Fig. 2,
- \(R_{MAX}\) – unit normal vector

Fig. 3. Normal force characteristic
The rolling friction force is a linear function of the pressure force.
\[ F_T = \mu |R_N|e_T \]  \hspace{1cm} (7)

where:
- \( \mu \) – rolling friction coefficient,
- \( e_T \) – unit tangential vector.

The unit tangential vector is defined as:
\[ e_T = \frac{V_T}{|V_T|} \]  \hspace{1cm} (8)

The damping force was introduced to the model to take into account the kinetic energy loss caused by the deformation of constructional elements of the module during collision. It was assumed in the presented model that the damping force is proportional to the normal velocity during collision:
\[ R_{DAMP} = -c_{DAMP} V_N \]  \hspace{1cm} (9)

where:
- \( c_{DAMP} \) – damping coefficient,
- \( V_N \) – normal velocity vector.

**The added water mass**

When a body submerged in water is subject to the action of a force, its inertia seems to be larger than in air. This effect is caused by the presence of the so called added water mass (added mass), i.e. the additional mass of water which moves together with the submerged body. When this body is accelerated the added water is also subject to acceleration, therefore in order to reach a given speed more momentum is to be delivered than when the body is not submerged.

The equation of linear motion in 1D can be written in the form:
\[ \frac{d}{dt} [(M + m)V] = F \]  \hspace{1cm} (10)

where:
- \( V \) – velocity,
- \( F \) – force,
- \( M \)– mass of submerged body,
- \( m \) – added water mass.

For 3D motion, the added mass is different for each of main directions, therefore the equations of linear motion for a 3D case can be written as:
\[ \frac{d}{dt} [(M + A_{11})V_x] = F_x \]  \hspace{1cm} (11)
\[ \frac{d}{dt} [(M + A_{22})V_y] = F_y \]  \hspace{1cm} (11)
\[ \frac{d}{dt} [(M + A_{33})V_z] = F_z \]

where:
- \( A_{11}, A_{22}, A_{33} \) – added masses in x, y and z direction, respectively.

A similar phenomenon can be observed for the angular motion. A submerged body which is subject to the action of a moment reveals larger moments of inertia than the same body when taken out of water. For a simple 1D case of angular motion, the dynamic equation has the form:
\[ \frac{d}{dt} [(I_{zz} + A_{44})\omega_z] = M_z \]  \hspace{1cm} (12)

The angular motion in 3D is described by the equations:
\[ (I_{xx0} + A_{44}) \frac{d}{dt} \omega_{x_0} + (I_{yy0} + A_{55}) \frac{d}{dt} \omega_{y_0} + A_{66} \frac{d}{dt} \omega_{z_0} = M_{x_0} \]  \hspace{1cm} (13)
\[ (I_{yy0} + A_{55}) \frac{d}{dt} \omega_{x_0} + (I_{zz0} + A_{44}) \frac{d}{dt} \omega_{y_0} + A_{66} \frac{d}{dt} \omega_{z_0} = M_{y_0} \]  \hspace{1cm} (13)
\[ (I_{zz0} + A_{66}) \frac{d}{dt} \omega_{x_0} + (I_{xx0} + A_{44}) \frac{d}{dt} \omega_{y_0} + A_{55} \frac{d}{dt} \omega_{z_0} = M_{z_0} \]  \hspace{1cm} (13)

where:
- \( A_{44}, A_{55}, A_{66} \) – added masses (or added moments of inertia) for the rotation around the \( x_{i0}, y_{0} \) and \( z_{0} \) axis, respectively.

The complete equations of angular motion in three degrees of freedom also include terms with \( A_{ij} \), where \( i \neq j \). The above presented form is simplified.

The presence of the added masses considerably affects characteristics of motion of a ship (or boat). Oscillation time periods are longer, while ship responses to external excitations are smaller, and its behaviour on wave is more stable. In the case of boat launching, the added water masses remarkably reduce its velocity. The phenomenon of slamming can be interpreted as non-elastic collision with masses of water which “join” the hull of the boat (module), thus becoming its added mass. When the added mass is taken into account in the equations, then the hydrodynamic force caused by slamming can be calculated using the formula given in [2].

**Modelling of the slaming phenomenon**

As written above, the slamming phenomenon can be described and its intensity can be predicted using the concept of added water mass. An elementary problem of calculating hydrodynamic reactions caused by body collision with water surface (in 2D) was discussed by von Karman in 1929. The mathematical model used by the authors bases on the concept of added mass changes.
Let us consider the following case: a body with mass \( M \) and added mass \( m \) bumps into water with velocity \( V_0 \), while the velocity of water is \( V_w \). After a short time interval \( \Delta t \) the added water mass increases to \( m + \Delta m \) and the velocity of the body changes to \( V_1 \). Based on the principle of conservation of momentum we have:

\[
(M + m)V_0 + \Delta mV_w = (M + m + \Delta m)V_1 \quad (14)
\]

The velocity of the body \( V_1 \) after the time \( \Delta t \) is:

\[
V_1 = \frac{(M + m)V_0 + \Delta mV_w}{M + m + \Delta m} \quad (15)
\]

and the average body acceleration in this interval:

\[
a = \frac{V_1 - V_0}{\Delta t} \quad (16)
\]

Hence, the average hydrodynamic reaction caused by the slamming effect is:

\[
F_{slam} = Ma \quad (17)
\]

It was assumed in the adopted model that the amount of the added mass is proportional to the area of the wetted surface \( S_{wet} \):

\[
m(S_{wet}) = \frac{S_{wet}}{S_0}m_0 \quad (18)
\]

where:

- \( m_{S_{wet}} \) – instantaneous added mass,
- \( S_0 \) – wetted body surface in hydrodynamic equilibrium state,
- \( m_0 \) – added mass in the state of hydrostatic equilibrium (when \( S_{wet} = S_0 \)).

The forces generated by slamming are calculated in successive time steps within the given time interval \( \Delta t \).

The above scheme refers to the 1D case, but it can be easily extended to 3D cases.

When the boat has some angular velocity before it comes into contact with water, then the hydrodynamic reaction caused by the moment of inertia of added masses appears. Applying the reasoning similar to that presented above, we can formulate the following formula for the moment of this reaction:

\[
M_{slam} = -\frac{\Delta I_{\omega_{boat}}}{\Delta t} \quad (19)
\]

where:

- \( M_{slam} \) – moment of reaction caused by slamming,
- \( \Delta I_{\omega_{boat}} \) – increment of the moment of inertia of the added mass,
- \( \omega_{boat} \) – angular velocity of the boat (module),
- \( \Delta t \) – time step.

The hydrodynamic forces caused by the slamming phenomenon only occur when the boat bumps into water \( \Delta m > 0 \), as only in this case the effect of collision between the mass of the boat and that of the water, with different initial velocities, takes place. When the boat emerges from water \( \Delta m < 0 \) we can also observe some sort of hydrodynamic reactions but they are generated by other phenomena.

The added masses of water are calculated using the strip theory.

**Wave – theoretical model**

In the presented method a linear model of wave was adopted. The wave was modelled as a sum of regular waves, i.e. independent sequences of elementary waves with given amplitudes, angular frequencies and phase shifts.

The velocity potential for the regular wave in deep water is given by the following equation:

\[
\Phi(x,z,t) = \frac{ag}{\omega} e^{ikx} \sin[kx - \omega(t - t_0)] \quad (20)
\]

where:

- \( a \) – wave amplitude,
- \( \omega \) – angular frequency,
- \( g \) – acceleration of gravity,
- \( k \) – wave number,
- \( t \) – time,
- \( t_0 \) – phase shift,
- \( x, y \) – coordinates of the point at which the velocity potential is calculated.

The dynamic boundary condition on free water surface is:

\[
\zeta(x, t) = -\frac{1}{g} \frac{\partial \Phi}{\partial t} \quad (21)
\]

From Equation (20) and the dynamic boundary condition (21) we obtain the z-coordinate of the wave profile:

\[
\zeta(x, t) = a\cos[kx - \omega(t - t_0)] \quad (22)
\]

The phase velocity and the dispersion relation are determined from the kinetic boundary condition on free surface for the vertical coordinate of the circular velocity:

\[
\frac{\partial \zeta}{\partial t} = \frac{\partial \Phi}{\partial \zeta} \quad (23)
\]

Then the dispersion relation (for deep water) can be derived:

\[
\omega^2 = kg \quad (24)
\]

The phase velocity of the wave is given by:

\[
C = \frac{g}{\omega} \quad (25)
\]

The horizontal and vertical velocity component can be calculated by integrating Equation (20) with respect to \( x \) and \( z \), respectively:

\[
u(x,z,t) = \frac{ag}{\omega} e^{ikx} \cos[kx - \omega(t - t_0)] \quad (26)
\]

\[
w(x,z,t) = \frac{ag}{\omega} e^{ikx} \sin[kx - \omega(t - t_0)] \quad (27)
\]

The hydrodynamic pressure is calculated from the Bernoulli equation:

\[
-\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} + gz = 0 \quad (28)
\]

after placing into it the velocity potential formula given by Equation (20). Then we get:

\[
p(x, y, z) = \rho a e^{ikx} \cos[kx - \omega(t - t_0)] \quad (29)
\]

Equation (20) can be modified in order to model the wave moving in an arbitrary direction:

\[
\Phi(x,y,z,t) = \sum_{i=0}^{n} a_i \frac{g}{\omega_i} e^{ik_i} \sin[k_i(x+e_{ix}y+e_{iy}z) - \omega_i(t - t_{0_i})] \quad (30)
\]

where:

- \( i \) – wave component index
- \( e_{ix}, e_{iy} \) – unit vector components which indicate the wave direction (in the present model the components \( e_{ix}, e_{iy} \) are the same for angular frequencies).
PRELIMINARY RESULTS OF TEST CALCULATIONS

Data for calculations

Figure 5 below shows a scheme of the modelled system.

![Diagram of system: ramp, module, pontoon and water](image)

**Fig. 5. The system: ramp, module, pontoon and water**

**Pontoon:**

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>Breadth</td>
<td>B</td>
<td>m</td>
</tr>
<tr>
<td>Height</td>
<td>H</td>
<td>m</td>
</tr>
<tr>
<td>Total volume</td>
<td>V</td>
<td>m^3</td>
</tr>
<tr>
<td>Mass</td>
<td>M_p</td>
<td>kg</td>
</tr>
<tr>
<td>Moment of inertia around x_0-axis</td>
<td>I_{xx0}</td>
<td>kg m^2</td>
</tr>
<tr>
<td>Moment of inertia around y_0-axis</td>
<td>I_{yy0}</td>
<td>kg m^2</td>
</tr>
<tr>
<td>Moment of inertia around z_0-axis</td>
<td>I_{zz0}</td>
<td>kg m^2</td>
</tr>
</tbody>
</table>

It was assumed that the mass centre is equivalent with the geometry centre.

The moments of inertia $I_{xx0}$, $I_{yy0}$ and $I_{zz0}$ were calculated using the formulas:

\[ I_{xx0} = \frac{1}{12} M_p (B^2 + H^2) \]  \hspace{1cm} (31)

\[ I_{yy0} = \frac{1}{12} M_p (L^2 + H^2) \]  \hspace{1cm} (32)

\[ I_{zz0} = \frac{1}{12} M_p (L^2 + B^2) \]  \hspace{1cm} (33)

**Ramp (without a pontoon):**

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>Breadth</td>
<td>B</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>M_r</td>
<td>kg</td>
</tr>
<tr>
<td>Moment of inertia around x_r-axis</td>
<td>I_{x0}</td>
<td>kg m^2</td>
</tr>
<tr>
<td>Moment of inertia around y_r-axis</td>
<td>I_{y0}</td>
<td>kg m^2</td>
</tr>
<tr>
<td>Moment of inertia around z_r-axis</td>
<td>I_{z0}</td>
<td>kg m^2</td>
</tr>
<tr>
<td>Maximum angle (from horizontal position)</td>
<td>$\alpha$</td>
<td>deg</td>
</tr>
</tbody>
</table>

**Module:**

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>Breadth</td>
<td>B</td>
<td>m</td>
</tr>
<tr>
<td>Height</td>
<td>H</td>
<td>m</td>
</tr>
<tr>
<td>Draught</td>
<td>T</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>M_m</td>
<td>kg</td>
</tr>
<tr>
<td>Longitudinal coordinate of mass centre</td>
<td>$x_{G0}$</td>
<td>m</td>
</tr>
<tr>
<td>Vertical coordinate of mass centre</td>
<td>$z_{G0}$</td>
<td>m</td>
</tr>
<tr>
<td>Moment of inertia around x_0-axis</td>
<td>I_{xx0}</td>
<td>kg m^2</td>
</tr>
<tr>
<td>Moment of inertia around y_0-axis</td>
<td>I_{yy0}</td>
<td>kg m^2</td>
</tr>
<tr>
<td>Moment of inertia around z_0-axis</td>
<td>I_{zz0}</td>
<td>kg m^2</td>
</tr>
</tbody>
</table>

where: $I_{xx0}$, $I_{yy0}$, $I_{zz0}$ were calculated using the formulas [7]:

\[ I_{xx0} = (0.3B)M_m \]  \hspace{1cm} (34)

\[ I_{yy0} = I_{zz0} = (0.225L)M_m \]  \hspace{1cm} (35)

**Sample results of calculations of launching in still sea**

**Scenario no 1**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of ramp fixing</td>
<td>H_{RF}</td>
<td>m</td>
</tr>
<tr>
<td>Ramp angle at t=0</td>
<td>$\alpha_0$</td>
<td>deg</td>
</tr>
<tr>
<td>Wave height</td>
<td>H_w</td>
<td>m</td>
</tr>
<tr>
<td>Wave period</td>
<td>T_w</td>
<td>s</td>
</tr>
<tr>
<td>Wave phase shift</td>
<td>t_{\phi}</td>
<td>s</td>
</tr>
</tbody>
</table>

![Boat trajectory del t = 0.2 s](image)

**Fig. 6. Module and ramp contours in time, $\Delta t = 0.2$ s**
CONCLUSIONS

The instantaneous positions of the outlines of the ramp and the module sliding from the ship on still water shown in Fig. 6 look very reliably. Also the time-histories of changes of basic velocity and acceleration components of the mass centre of the launched module shown in Fig. 7 and Fig. 8 are consistent with the experience and expectations of the authors, which testifies to the correctness of the developed model and numerical code.

The developed method also makes it possible to perform numerical simulations for various scenarios of sea conditions. The obtained results of these simulations will be presented and discussed in a separate publication.

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INTRODUCTION

Triangular meshing is achieved with algorithms which use either Delaunay Triangulation (DT) presented in [2] and [6] or Constrained Delaunay Triangulation (CDT) presented in [3]. Empty circumcircle condition of Delaunay Triangulation ensures best shaped triangles on a given set of vertices. Also mesh refinement algorithms as in [10], which improve mesh by inserting new vertices to break triangles having excessive angles hold empty circumcircle condition. Checking in two-dimensional space if there is any other vertex within circle circumscribed on any mesh triangle, is not difficult. However it requires precise calculations due to round-off vulnerability of floating math – see [11]. Provided that points A, B, C are in the counter-clockwise order point D lies within ABC circumcircle when condition presented in formula (1.1) is satisfied.

\[
\begin{bmatrix}
    x_A - x_D & y_A - y_D & (x_A - x_D)^2 + (y_A - y_D)^2 \\
    x_B - x_D & y_B - y_D & (x_B - x_D)^2 + (y_B - y_D)^2 \\
    x_C - x_D & y_C - y_D & (x_C - x_D)^2 + (y_C - y_D)^2
\end{bmatrix} < 0
\]  

(1.1)

In our considerations we assume the surface to be described as NURBS (Non-Uniform Rational B-Spline) surface. A NURBS surface of degree p in u direction and degree q in v direction is a bivariate vector-valued function described by the formula (1.2) as in [5] and [9].

\[
S(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} R_{i,j}(u, v)P_{i,j}
\]  

(1.2)

The \{P_{i,j}\} is a bidirectional net of control points and \(R_{i,j}(u, v)\) is a piecewise rational basis function as in formula (1.3).

\[
R_{i,j}(u, v) = \frac{\sum_{k=0}^{n} \sum_{l=0}^{m} N_{k,p}(u)N_{l,q}(v)\sigma_{k,l}}{\sum_{k=0}^{n} \sum_{l=0}^{m} N_{k,p}(u)N_{l,q}(v)}
\]  

(1.3)

The \{\sigma_{k,l}\} are the weights while \{N_{k,p}(u)\} and \{N_{l,q}(v)\} are non-rational B-spline basis functions defined on the knot vectors:

\[
U = \{0, ..., 0, u_{p+1}, ..., u_{r-p-1}, 1, ..., 1\}
\]

\[
V = \{0, ..., 0, v_{q+1}, ..., v_{s-q-1}, 1, ..., 1\}
\]

Formula (1.2) and (1.3) is a parametric form of the NURBS surface. Parameters \(u\) and \(v\) define parametric space for the surface which is a plane. Vector-value function returns a vector of co-ordinates \([x, y, z]\) – point on a surface, for given \((u, v)\) pair of parameters. If a surface was developable it should be enough to triangulate it in parameter space (on a plane). Then the result triangles in 3D space should have the same properties as in the parameter space. However the surface may be curved in such a way that transformation from parametric space do 3D space causes distortion which changes length...
and angles. This is the reason why it is not possible to mesh only in the parametric space. To properly mesh the surface the triangulation must be denser in the regions where the distortion is more significant.

**NURBS CURVATURE**

To be able to identify regions where the triangular mesh is to be denser it is necessary to define some measure of distortion in mapping between parametric space and the surface. The simplest measure is curvature. The curvature is defined for parametric curve \( C \) as its second derivative with respect to the curve length - see formula (2.1) as in [5].

\[
k = k(s) = \frac{dC(s)}{ds} = C''(s) \quad (2.1)
\]

In Euclidean three-dimensional space there are two curvatures defined for a parametric surface \( S(u, v) \): Gaussian curvature and mean curvature. Both requires finding two principal directions on the surface - the surface curves with maximum and minimum curvatures. Hence to compute a surface curvature at \( (u, v) \) it is necessary to select two surface curves from all surface curves crossing this point: one with maximum curvature \( k_1 \) and one with minimum curvature \( k_2 \). The Gaussian curvature is the product of \( k_1 \) and \( k_2 \) – see formula (2.2) as in [1].

\[
K = k_1 \cdot k_2 \quad (2.2)
\]

The mean curvature is the mean value of \( k_1 \) and \( k_2 \) – see formula (2.3) as in [1].

\[
H = \frac{k_1 + k_2}{2} \quad (2.3)
\]

In case of general parametric surface \( S(u, v) \) finding the principal directions might be computationally expensive.

**TANGENT PLANE AND DISTANCE**

Triangular meshing of a surface is a method of surface approximation. The maximum distance between a point on the surface and its approximating mesh seems to be a good measure of mesh quality. It seems even better than finding the maximum curvature point – see Fig. 1.

\[
\text{Fig. 1. Maximum distance and maximum curvature points}
\]

In [8] Laug proposed two measures: distance deviation \( A_0 \) and angle deviation (difference in a normal vector angle) \( A_1 \). However he did not give any suggestions how to compute the measures and how expensive the calculations are. He focused mainly on discussing when to apply each of the measures to obtain pertinent results.

Finding maximum distance for the entire mesh requires checking every mesh triangle. To assess the quality of a single mesh triangle it is necessary to find a plane tangent to the surface, which is parallel to the triangle. When a point of tangency lies between vertices of the triangle in the parametric space then the distance between the parallel plane and the triangle is the maximum distance between a point on the surface and the triangle – see Fig. 2.

\[
\text{Fig. 2. A mesh triangle and the parallel plane tangent to the surface}
\]

**OPTIMIZATION PROBLEM**

Finding the parallel tangent plane requires solving a system of two equations presented in formula (4.1). Vector \( N \) is known as we know three vertices \( P_1, P_2, P_3 \) of the mesh triangle, which also belong to the surface \( S \).

\[
N \cdot S'_u(u, v) \quad (4.1) \\
N \cdot S'_v(u, v)
\]

where:

\[
N = P_1P_2 \times P_1P_3 \\
S'_u(u, v) \quad \text{the first derivative with respect to } u \\
S'_v(u, v) \quad \text{the first derivative with respect to } v
\]

Geometric interpretation of the formula (4.1) is as follows: the plane tangent to the surface \( S \) is defined by two partial derivatives: \( S'_u \) with respect to parameter \( u \), and \( S'_v \) with respect to parameter \( v \). Each partial derivative is a vector tangent to the surface \( S \) in the same point but in different direction. If vector \( N \) is perpendicular to the plane tangent to the surface \( S \), then its dot product with each partial derivative \( S'_u \) and \( S'_v \) must be zero.

Solution of the system of equations (4.1) is a pair of parameters \( (u, v) \). With these parameters we obtain a point \( P = S(u, v) \), which is a point of tangency. The distance between the point \( P \) and the triangle \( P_1P_2P_3 \) shows how close is the mesh triangle to the surface. The closer the point the better the quality of the triangle and vice versa.

There might be more than one solution for the system of two equations presented in formula (4.1). It is due to the fact that the surface is a free-form surface of any shape. Finding more than one solution is a natural indication for making triangular mesh denser.

**OPTIMIZATION ALGORITHM**

To solve the system of equations presented in formula (4.1) we use Newton iteration. The method is similar to that used to solve the point inversion problem presented in [9] and [4]. First we change each equation into the function as in formula (5.1)

\[
f(u, v) = N \cdot S'_u(u, v) = 0 \\
g(u, v) = N \cdot S'_v(u, v) = 0 \quad (5.1)
\]
Iterations require to change the values of parameters $u$ and $v$.

\[
\begin{align*}
  u_{i+1} &= u_i - \Delta u \\
  v_{i+1} &= v_i - \Delta v
\end{align*}
\]  

(5.2)

where:

\[
\begin{align*}
  \Delta u &= \left[ \begin{array}{cc} f_u'(u_i, v_j) & f'_v(u_i, v_j) \\
                          g_u'(u_i, v_j) & g'_v(u_i, v_j) \end{array} \right]^{-1} \left[ \begin{array}{c}
                          f(u_i, v_j) \\
                          g(u_i, v_j) \end{array} \right] \\
  \Delta v &= \left[ \begin{array}{cc} f_u'(u_i, v_j) & f'_v(u_i, v_j) \\
                          g_u'(u_i, v_j) & g'_v(u_i, v_j) \end{array} \right]^{-1} \left[ \begin{array}{c}
                          f(u_i, v_j) \\
                          g(u_i, v_j) \end{array} \right]
\end{align*}
\]  

(5.3)

\[
\begin{align*}
  f_u'(u, v) &= N \cdot S_{uu}''(u, v) \\
  f'_v(u, v) &= N \cdot S_{uv}''(u, v) \\
  g_u'(u, v) &= N \cdot S_{uu}''(u, v) \\
  g'_v(u, v) &= N \cdot S_{uv}''(u, v)
\end{align*}
\]  

(5.4) - (5.7)

Iterations are finished when the convergence criterion (5.8) has been met or maximum number of iterations has been reached.

\[
\left| S(u_{i+1}, v_j) - S(u_{i-1}, v_j) \right| < \varepsilon
\]  

(5.8)

The found solution is the local extreme. As it should not take many iterations it is possible to start searching for the solution from each of the vertices of the mesh triangle. If the result is the same from each vertex it is possible that the solution found is the only solution.

The Newton iteration has been chosen to find solution because it involves relatively not expensive calculations: i.e. the first and second partial derivatives and matrix multiplication. It is also recognized capable of to bringing most reliable results in other fundamental calculations performed on NURBS e.g. point inversion. To find the maximum distance between a mesh triangle and the surface one could also use random search methods. It would simplify calculations, however it would require more iterations as the process of choosing subsequent solutions is of a stochastic nature.

**EXPERIMENT RESULTS**

The NURBS surface of second degree (quadratic) has been taken to experiment. Tab. 1 lists control points of the surface. The shape of the NURBS surface is presented with green color in Fig. 3. The rectangular net of control points is shown above the surface.

Six triangles with vertices on the surface have been chosen to check the tangent plane search algorithm presented in the previous chapter. For each triangle a parallel plane tangent to the surface has been found and a distance between a plane and a triangle has been calculated. The results are presented in Tab. 2.

Each triangle vertex in Tab. 2 is described both in three-dimensional space with the co-ordinates $(x, y, z)$ and in parametric space of the NURBS surface with the pair of parameters $(u, v)$.

<table>
<thead>
<tr>
<th>No</th>
<th>1-st vertex</th>
<th>2-nd vertex</th>
<th>3-rd vertex</th>
<th>Normal vector</th>
<th>Tangency point</th>
<th>Distance iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(11.02, 14.23, 9.48) (0.33, 0.33)</td>
<td>(11.02, 25.41, 9.61) (0.33, 0.66)</td>
<td>(18.71, 20.00, 10.64) (0.66, 0.50)</td>
<td>(12.25, 0.98, -85.952)</td>
<td>(14.20, 19.81, 11.61)</td>
<td>1.593 2</td>
</tr>
<tr>
<td>2</td>
<td>(13.95, 18.33, 11.48) (0.45, 0.45)</td>
<td>(13.95, 21.67, 11.48) (0.45, 0.55)</td>
<td>(16.05, 20.00, 11.57) (0.55, 0.50)</td>
<td>(0.28, 0.00, -7.00)</td>
<td>(14.79, 20.00, 11.66)</td>
<td>0.146 2</td>
</tr>
<tr>
<td>3</td>
<td>(3.80, 13.13, 4.13) (0.10, 0.30)</td>
<td>(3.80, 16.67, 4.93) (0.10, 0.40)</td>
<td>(12.80, 14.95, 10.47) (0.40, 0.35)</td>
<td>(21.00, 7.26, -31.88)</td>
<td>(8.87, 16.21, 8.81)</td>
<td>0.530 3</td>
</tr>
<tr>
<td>4</td>
<td>(7.20, 13.13, 6.75) (0.20, 0.30)</td>
<td>(7.20, 16.67, 7.73) (0.20, 0.40)</td>
<td>(12.80, 14.95, 10.47) (0.40, 0.35)</td>
<td>(11.39, 5.51, -19.83)</td>
<td>(10.11, 15.51, 9.41)</td>
<td>0.271 4</td>
</tr>
<tr>
<td>5</td>
<td>(7.20, 9.17, 5.13) (0.20, 0.20)</td>
<td>(12.80, 9.17, 7.53) (0.40 0.20)</td>
<td>(10.20, 30.83, 6.63) (0.30 0.80)</td>
<td>(-52.00, -1.20, 121.33)</td>
<td>(11.90, 19.84, 10.92)</td>
<td>3.372 2</td>
</tr>
<tr>
<td>6</td>
<td>(7.20, 9.17, 5.13) (0.20 0.20)</td>
<td>(12.80, 9.17, 7.53) (0.40 0.20)</td>
<td>(10.20, 16.67, 9.73) (0.30 0.40)</td>
<td>(-18.00, -18.56, 42.00)</td>
<td>(11.58, 13.04, 9.26)</td>
<td>0.455 4</td>
</tr>
</tbody>
</table>

*Tab. 1. Control points {P}_{ij} of the NURBS surface*

*Tab. 2. Tangent plane search results for the six triangles on the NURBS surface*

The surface knot vector in $u$ direction: $U = \{0.0, 0.0, 0.0, 0.5, 1.0, 1.0, 1.0\}$

The surface knot vector in $v$ direction: $V = \{0.0, 0.0, 0.0, 0.4, 0.6, 1.0, 1.0, 1.0\}$
SUMMARY

The presented optimization method does not guarantee to always find a plane parallel to the mesh triangle and tangent to the NURBS surface. Especially, when the NURBS surface has inflection points and is partially convex and partially concave there might be many solutions. With Newton iteration only the solution closest to the point from which the search started, is found.

However the distance between a parallel tangent plane and a mesh triangle is a good measure of accurate meshing. The experiment results presented in Tab. 2 e.g. in line 1 and 2 show that the smaller triangle in line 2 is about ten times closer to the surface than the bigger triangle in line 1. Both tests required only two iterations – a very quick convergence. Next pairs of lines in Tab. 2, e.g. 3 and 4 or 5 and 6, also shows significant reduction of the distance between the NURBS surface and mesh triangles with reduction of the size of a triangle. Quick convergence of the search method results in the short execution time. The presented method seems to be promising and may be applied even if every triangle of the meshed surface was to be checked in this way.

Advantages of the presented method:
- distance between a mesh triangle and a parallel plane tangent to the NURBS surface is a good measure of mesh quality,
- quick convergence – several iterations to find solution,
- short execution time due to quick convergence and not complicated operations (NURBS surface derivatives)

An alternative way to assess mesh quality would be calculation of volume contained between the NURBS surface and a mesh triangle plane. However volume calculation requires calculating integrals of the NURBS surface, which is more complicated and time consuming than calculating derivatives. The comparison of the two measures could be interesting although it is out of the scope of this paper.

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A model for service life control of selected device systems

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ABSTRACT

This paper presents a way of determining distribution of limit state exceedence time by a diagnostic parameter which determines accuracy of maintaining zero state. For calculations it was assumed that the diagnostic parameter is deviation from nominal value (zero state). Change of deviation value occurs as a result of destructive processes which occur during service. For estimation of deviation increasing rate in probabilistic sense, was used a difference equation from which, after transformation, Fokker-Planck differential equation was obtained [4, 11]. A particular solution of the equation is deviation increasing rate density function which was used for determining exceedence probability of limit state. The so-determined probability was then used to determine density function of limit state exceedance time, by increasing deviation. Having at disposal the density function of limit state exceedance time one determined service life of a system of maladjustment. In the end, a numerical example based on operational data of selected aircraft [weapon] sights was presented. The elaborated method can be also applied to determining residual life of shipboard devices whose technical state is determined on the basis of analysis of values of diagnostic parameters.

Keywords: reliability; unreliability; durability; service life time; destructive processes

INTRODUCTION

Many devices are equipped with systems, mechanisms which are subject to regulation processes and as a result of them the system is brought back to a required state (its nominal values). Application of an “in-service correction” to a device results from occurrence of its maladjustment. The maladjustment is caused by degradation and ageing processes taking place during device operation. Information about occurrence of such processes comes from diagnostic parameters which change their values under influence of destructive processes [7, 8, 13]. Certain types of aircraft [weapon] sights exemplify the devices subject to the above mentioned processes. Their task is a. o. to elaborate and display sight information in the form of a sight marker. The devices are required a. o. to maintain, within certain limits, zero state regarding determined sight and advance angles.

It is assumed that deviations from zero state are identified by means of determinate diagnostic parameters. When values of the diagnostic parameters exceed limit value then the device becomes not fully serviceable, i. e. it reaches a state of intermediate serviceability. In this case it is necessary to regulate the device and bring its indications to zero position. In this paper is presented a model of the determining of time interval during which quantity of a parameter increases from its initial value to limit one. The presented method makes it possible to ensure a greater working accuracy of a device and also to control process of its operation.

DETERMINATION OF DENSITY FUNCTION OF DIAGNOSTIC PARAMETER CHANGES

In the proposed model of service life estimation the following assumptions have been taken:

• Technical state of a device is determined by only one diagnostic parameter: \( z \) in the form of deviation from initial state (zero value).

\[
 z = |X - X_{nom} | \tag{1}
\]

where:

\( X \) - current value of diagnostic parameter;
\( X_{nom} \) - nominal value (zero state) of diagnostic parameter.

• Change in value of diagnostic parameter deviation occurs during entire service time (operation and standstill);

• The parameter \( z \) is non-decreasing;

• Rate of change of diagnostic parameter can be described by the following relation:

\[
 \frac{dz}{dt} = c \tag{2}
\]

where:

\( c \) - random variable which characterizes susceptibility of an element to changes dependent on its features and working conditions;
\( t \) - calendar time.
Dynamics of changes of the deflection \( z \) is characterized, in probabilistic sense, by the following difference equation:

\[
U_{z,t} + \Delta t = PU_{z - \Delta z,t} \tag{3}
\]

where:
- \( U_{z,t} \) - probability of the event that in the instant \( t \) diagnostic parameter quantity takes the value of \( z \);
- \( P \) - probability of the event that in the time interval \( \Delta t \) deflection value will increase by the value \( \Delta z \). It is assumed that \( P = 1 \);
- \( \Delta z \) - deflection increase.

In the functional form Eq. (3) is as follows:

\[
U(z, t + \Delta t) = U(z, t) + \frac{\partial U(z, t)}{\partial t} \Delta t \tag{4}
\]

Eq. (4) has the following sense: probability of that in the instant \( t \) deflection value was equal to \( z - \Delta z \) and in the time interval \( \Delta t \) it increased by the value \( \Delta z \).

Eq. (4) is now transformed into a partial differential equation. To this end the following approximations are assumed:

\[
\frac{\partial U(z, t)}{\partial t} = -b \frac{\partial U(z, t)}{\partial z} + \frac{1}{2} \frac{\partial^2 U(z, t)}{\partial z^2} \tag{5}
\]

With the help of Eq. (5), Eq. (4) is transformed to the following form:

\[
\frac{\partial U(z, t)}{\partial t} = -b \frac{\partial U(z, t)}{\partial z} + \frac{1}{2} \frac{\partial^2 U(z, t)}{\partial z^2} \tag{6}
\]

where:
- \( b = E[c] \) - mean increase of diagnostic parameter deflection value per unit of time;
- \( a = E[c^2] \) - mean square of increase of diagnostic parameter deflection value per unit of time.

Solution of Eq. (6) takes the form \([1, 14]\):

\[
u(z, t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(z - b t)^2}{2A(t)}} \tag{7}
\]

where:
- \( B(t) = \int_0^t b dt = bt \)
- \( A(t) = \int_0^t a dt = at \)

The density function of increase of diagnostic parameter deflection value can be directly applied to estimation of reliability of a device’s system.

**DETERMINATION OF DISTRIBUTION OF TIME OF EXCEEDANCE OF LIMIT (PERMISSIBLE) STATE**

The probability of exceedance of limit state by diagnostic parameter can be expressed with the help of the density function of changes of diagnostic parameter deflection, (7), as follows:

\[
Q(t; z_g) = \int_{z_g}^{\infty} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z - bt)^2}{2at}} dz \tag{8}
\]

The distribution density function of time of the first exceedance over the permissible value \( z_g \) takes the following form:

\[
f(t) = \frac{\partial}{\partial t} Q(t; z_g) \tag{9}
\]

Taking into account Eq. (7) one obtains:

\[
f(t) = \frac{\partial}{\partial t} \int_{z_g}^{\infty} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z - bt)^2}{2at}} dz \tag{10}
\]

Therefore,

\[
f(t) = \int_{z_g}^{\infty} \left[ \frac{\partial}{\partial t} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z - bt)^2}{2at}} \right] dz \tag{11}
\]

Using the formulation (7) one gets:

\[
f(t) = \int_{z_g}^{\infty} \left[ \frac{\partial}{\partial t} \nu(z, t) \right] dz \tag{12}
\]

Moreover, the time derivative of the function of the form:

\[
\frac{\partial}{\partial t} \left[ u(z, t) \right] = u(z, t) \left( \frac{z^2 - b^2 t^2 - at}{2at^2} \right) \tag{13}
\]

The relation (13) is now inserted into Eq. (11) to obtain the following:

\[
f(t) = \int_{z_g}^{\infty} \left[ \frac{\partial}{\partial t} u(z, t) \right] dz \tag{14}
\]

Then, an anti-derivative function of the integrand of Eq. (14) is searched for. It is expected that the function of the form:

\[
w(z, t) = u(z, t) \left( \frac{z + bt}{2t} \right) \tag{15}
\]

is the searched anti-derivative for the integrand of Eq. (14).

The test of this hypothesis is now performed:

\[
\frac{\partial}{\partial z} \left[ u(z, t) \left( \frac{z + bt}{2t} \right) \right] = u(z, t) \left( -\frac{z - bt}{at} \right) \left( \frac{z + bt}{2t} \right) + u(z, t) \left( -\frac{1}{2t} \right) \tag{16}
\]

It can be concluded that the anti-derivative for the integrand of Eq. (14) is of the following form:

\[
w(z, t) = u(z, t) \left( \frac{z + bt}{2t} \right) \tag{17}
\]

Now the integral (14) is calculated to get:

\[
f(t) = u(z, t) \left( \frac{z + bt}{2t} \right) \bigg|_{z_g}^{\infty} \tag{18}
\]
The relation (17) determines the distribution density function of time of the first exceedance of limit (permissible) state by diagnostic parameter deflection.

**ESTIMATION OF SERVICE LIFE OF SELECTED STRUCTURAL SYSTEMS OF SEA-GOING SHIP OR AIRCRAFT**

The formula for reliability of a ship or aircraft equipment system takes the following form:

$$ R(t) = 1 - \int_{0}^{t} f(t) \, dt \quad (18) $$

where the density function $f(t)$ is determined by Eq. (17).

However, unreliability of a ship or aircraft equipment system can be determined by using Eq. (19) as follows:

$$ Q(t) = \int_{0}^{t} f(t) \, dt \quad (19) $$

The integral (19) should be transformed to a simpler form. It may be observed that the integrand can be written in the form:

$$ f(t) = \frac{z_{g} + bt}{2t} \cdot \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z_{g} - bt)^2}{2at}} \quad (20) $$

to reduce the problem to the indefinite integral:

$$ \int \frac{(z_{g} + bt)}{2t} \cdot \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z_{g} - bt)^2}{2at}} \, dt \quad (21) $$

On substitution of:

$$ \frac{(bt - z_{g})^2}{2at} = u $$

the integral (21) takes the form:

$$ \int \frac{z_{g} + bt}{2t} \cdot \frac{1}{\sqrt{2\pi at}} e^{-\frac{(bt - z_{g})^2}{2at}} \, du = \int \frac{1}{2\sqrt{\pi}} \cdot \frac{1}{\sqrt{u}} e^{-u} \, du \quad (22) $$

Next, the subsequent substitution should be done: $\sqrt{u} = w$, $du = 2wdw$

Taking into account the above given relations one can write the integral (22) in the following form:

$$ \frac{1}{\sqrt{2\pi}} \int w e^{-w^2} \, dw = \frac{1}{\sqrt{\pi}} \int e^{-w^2} \, dw \quad (23) $$

On substitution of:

$$ w^2 = \frac{y^2}{2} $$

$$ dw = \frac{y}{\sqrt{2}} $$

the integral of the following form is achieved:

$$ \frac{1}{\sqrt{2\pi}} \int e^{-\frac{y^2}{2}} \, dy \quad (24) $$

where:

$$ y = \frac{bt - z_{g}}{\sqrt{at}} $$

Inserting the obtained results into Eq. (18) and not forgetting to assign appropriate integration limits one obtains the formula for the reliability in question:

$$ R(t) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{bt - z_{g}}{\sqrt{at}}} e^{-\frac{y^2}{2}} \, dy \quad (25) $$

The cumulative distribution function of normal standardized distribution takes the following form, [10, 12]:

$$ \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} \, dy \quad (26) $$

Taking into account the relation (26), one can finally express the formula for reliability of a ship or aircraft structural element as follows:

$$ R^*(t) = 1 - \Phi\left(\frac{bt - z_{g}}{\sqrt{at}}\right) \quad (27) $$

where $b^*$ and $a^*$ are coefficients of the values estimated on the basis of operational data of sea-going ships or aircraft.

Therefore risk of failure of a device can be determined by means of the following relation:

$$ Q^* = 1 - R^*(t) = \Phi(\gamma) \quad (28) $$

where:

$$ -\gamma = \frac{bt - z_{g}}{\sqrt{a^*t}} \quad (29) $$

After assuming an unreliability level, the upper limit of the integral, $\gamma^*$, in Eq. (28), can be determined. Having $\gamma^*$ from the relation (29) one can determine the service life of a device system as follows:

$$ T = -\frac{(2b^2z_{g} + (\gamma^*)^2a^*) - \sqrt{(2b^2z_{g} + (\gamma^*)^2a^*)^2 - 4b^2z_{g}^2}}{2b^2} \quad (30) $$

After the operational period $T$ the system undergoes regulation.

To make use of the formula (30), values of the constants appearing in it should be first determined (estimated). To this end, it is assumed that the data on increasing value of diagnostic parameter deflection, obtained from observation of a tested device during its operation process, are available in the form:

$$ [(z_{0}, t_{0}), (z_{1}, t_{1}), (z_{2}, t_{2}), ..., (z_{n}, t_{n})] \quad (31) $$

The best method for determining values of „$b^*$“ and „$a^*$“ from available data is that which makes use of likelihood function which can be generally expressed in the form as follows [3, 5, 18]:

$$ L = \prod_{k=0}^{n-1} g(t_{k}, z_{k}, \theta_1, \theta_2, ..., \theta_m) \quad (32) $$

where:

$$ g(t_{k}, z_{k}, \theta_1, \theta_2, ..., \theta_m) - \text{whole probability density function of the variable } z;$$
Therefore, once the data which describe diagnostic parameter values in the form of \([z_{01}, t_{01}], (z_{12}, t_{12}), \ldots, (z_{nT}, t_{nT})\) are at disposal, the values of density function coefficients can determined on the basis of Eq. (35) and (36):

\[
b^*_c = 0.042, \quad a^*_c = 0.011
\]  

After assuming the level of maintaining zero state, the value of the parameter \(\gamma^* = 2.35\) was read from normal distribution tables. Next, the parameter \(z_g\) was determined from technical documentation used for maintenance operations, where information on permissible values of deviations of diagnostic parameters was included.

As the values of the parameters \(b^*_c, a^*_c, \gamma^*, z_g\) have been already known, one inserted them into Eq. (30) to calculate the operation period after passing of which values of the diagnostic parameters will exceed limit state. For the case in question the period was found equal to:

\[
T_\varepsilon = 12[\text{months}]
\]  

The obtained value (38) may be used for technical maintenance operations depending on an assumed maintenance strategy. On the basis of the presented method it is possible to determine successive periods in which control of diagnostic parameter of a device should be done (Fig. 2) and regulation performed in order to bring the measured quantities to zero (nominal) position.

Fig. 2. Schematic diagram showing periods of device regulation to be performed to recover its nominal state

Summing up, it can be stated that the presented method seems to be correct and reasonable and makes it possible to perform technical state analysis of a device regarding character of changes of values of diagnostic parameters. The presented calculation example made it possible to verify the elaborated model as well as to reveal applicability merits of the method in question. The method may be useful in further efforts to improve both operation process and way of using sea-going ships or aircraft together with their on-board systems, by making it possible to determine the period during which the devices maintain state of serviceability.

Moreover, the presented method, in view of its general character, may be successfully applied to determination of residual service life of any technical object whose technical state can be determined on the basis of analysis of values of diagnostic parameters.

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Flow of liquid in flat gaps of the satellite motor working mechanism

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ABSTRACT

The article describes the methodology and results of investigations of the flow of oil and HFA-E emulsion in flat gaps of the working mechanism of a satellite motor. The flow of liquid in those gaps is turbulent and not fully developed. The article presents two methods of modelling this flow. Method I makes use of the Darcy-Weisbach formula, while Method II bases on the assumption that in the variable-length gaps the flow is turbulent in the area where the length is the smallest and laminar where the length is the largest. Consequently, the flow in such gaps can be modelled as the sum of laminar and turbulent flows. The results obtained in the experiment have made the basis for calculating relevant coefficients and assessing the proportion of turbulence in the flow modelled using both methods.

Keywords: flow in flat gap; satellite motor; HFA-E emulsion; oil

INTRODUCTION

For some years, the investigations of hydraulic satellite motors with small working volumes have been performed in the Department of Hydraulics and Pneumatics, Gdansk University of Technology [1,4-12]. The most recent design of the motor, worked out by the author, is shown in Fig. 1.

The working mechanism of the motor is a toothed unit (Figs. 1 and 2) consisting of: the toothed rotor R (4 teeth), the toothed curvature C (6 teeth), and the toothed wheels S (satellites). The number of satellites is equal to the sum of the numbers of rotor teeth and curvature teeth (i.e. 10 satellites in total).

The principle of operation of the satellite mechanism is the following. When the rotor rotates, the volume of the space between the satellites changes. This space is the working chamber. When its volume increases the filling cycle takes place, while when it decreases we have to deal with the emptying cycle. 24 cycles correspond to one shaft revolution. The chambers in the satellite mechanism are closed by the distribution plates (Fig. 1 - element 7 and 8, and Fig. 2), which also play a role of compensation plates. Thus the satellite motor has an ability to compensate axial clearances of the rotor and satellites.

The satellite motors reveal the smallest mass and overall dimensions as compared to other hydraulic motors. The power [kW] to mass [kg] ratio in those motors exceeds 4. Moreover, these motors can be fed with various liquids, including water.

The satellite motors can be widely used in the shipbuilding industry. For instance, they can be used as rudder drives, anchor...
hoists, and/or drives of various deck winches and davits. The motors with small working volumes are ideal drives for capstans and capstan winches on yachts (Fig. 3).

Mechanic, pressure and volumetric losses affect the efficiency of energy conversion in the hydraulic motor. It is the leakage flow through flat gaps in the working mechanism which affects the volumetric losses the most (Fig. 1 and Fig. 2). Results of past experimental investigations have made it possible to formulate a hypothesis that the non-fully developed turbulent flow takes place in these gaps. This is especially noticeable when the hydraulic motor is fed with low-viscosity medium, water or HFA-E emulsion for instance. The largest flow disturbance is observed in the areas where the gaps are the shortest, i.e. in the areas of teeth cooperation (Fig. 2).

The literature provides no mathematical models to describe flows in flat gaps with changing length, in particular the flow in the flat gaps of the satellite working mechanism. Publications can only be found which describe the volumetric losses in a global sense, without naming types of gaps [1, 2, 3, 11, 12]. Therefore, a detailed analysis of the flow in flat gaps of the working mechanism of the satellite motor is advisable, as it will provide opportunities for a more detailed simulation of volumetric loss characteristics in the hydraulic motor.

**METHODOLOGY OF EXAMINATION OF THE FLOW IN FLAT GAPS**

In order to determine experimentally the flow rate $Q_{fg}$ in flat gaps, we have to measure the absorbing power $Q$ at small constant velocity $n$ and at given pressure drop $\Delta p$ in the motor. Keeping the rotational speed of the motor low is only possible when it is coupled with the worm gear (Fig. 4). The speed of the examined motor is controlled using the electric motor with a frequency converter. The accumulator damps pressure pulsations in the system.

![Scheme of the measuring system: M – examined motor, P – pump, A – accumulator, E – electric motor with frequency converter, SV – safety valve, WG – worm gear, DT – experimental data recorded, Q – flow meter, FT – force sensor (for moment measurement), P1 and P2 – pressure sensors, T – temperature sensor, n1 – rotational speed sensor, AP – sensor measuring angular position of the shaft](image)

It is advisable to measure all parameters at very low and constant rotational speed of the shaft. As a result, the characteristic of instantaneous absorbing power $Q_m$ of the motor as a function of shaft rotation angle is obtained $\alpha$ (Fig. 5). The absorbing power $Q_m$ is the sum:  

$$Q_m = \frac{Q_{th} \cdot n + Q_{Lfgm} + Q_{Cm}}{Q_{Lmg}}$$  

(1)

where:

- $Q_{th}$ – theoretical working volume;
- $n$ – rotational speed of the shaft;
- $Q_{th}$ – theoretical absorbing power of the motor;
- $Q_{Lmg}$ – instantaneous volumetric loss;
- $Q_{Lfgm}$ – instantaneous flow rate in flat gaps;
- $Q_{Cm}$ – instantaneous flow rate in the gaps when inflow and outflow passages are closed by satellites (the flow in short gaps when the dead chamber is created).

We can assume that:

$$Q_{Lfgm} = Q_{Lfg}$$  

(2)

Where:

$Q_{Lfg}$ – the average flow rate in flat motor gaps.

Then:

$$Q_{Lfg} + Q_{th} = Q_{fg}$$  

(3)

The quantity $Q_{fg}$ can easily be assessed from experimental data (Fig. 5).
Since the rotational speed of the motor is very small, the pressure drop in inner motor passages can be neglected and:

\[ \Delta p_i = \Delta p = p_1 - p_2 \]  

(4)

The flow rate \( Q_{Lfg} \) in flat gaps is the sum:

\[ Q_{Lfg} = Q_{l1} + Q_{l2} \]  

(5)

where:
- \( Q_{l1} \) – flow rate from high pressure chambers to low pressure chambers;
- \( Q_{l2} \) – flow rate from high pressure chambers to shaft chamber.

From the shaft chamber the medium is passed to the low-pressure inner outflow chamber. Hence we can assume that:

\[ \Delta p_{i1} = \Delta p_{i2} = \Delta p_i \]  

(6)

where:
- \( \Delta p_{i1} \) – pressure drop in working chambers;
- \( \Delta p_{i2} \) – pressure drop in rotor gaps between high pressure chambers and the shaft chamber.

The motor examination was carried out at the rotational speed \( n = 1 \text{ rpm} \).

PARAMETERS OF EXAMINED MOTORS AND WORKING MEDIA

In the satellite motor the axial clearance \( h_R \) in the rotor is not equal to the axial clearances \( h_S \) of the satellites. In order to simplify the problem description, a definition of the equivalent assembly axial clearance is introduced \( h_0 \):

\[ h_0 = \frac{h_R + h_S}{2} \]  

(7)

In the loaded engine the gap height \( h \) is the function of pressure:

\[ h = h_0 - (D_c \cdot H_c + D_k) \cdot \Delta p_i \]  

(8)

where:
- \( D_k \) – constant dependent on rigidity of the axial clearance compensation unit;
- \( D_c \) – constant dependent on curvature rigidity;
- \( H_c \) – curvature height.

The constant \( D \) represents the average change of gap height caused by the pressure drop in the motor. The parameters \( D_k \) and \( D_c \) are determined from numerical calculations of deformation and dislocations of working mechanism and clearance compensation unit elements.

In the satellite working mechanism the shape of the flat gaps is irregular. Moreover, the width \( b \) and the length \( l \) of the gaps change during the operation of the mechanism. All this makes precise description of their dimensions impossible. Therefore an assumption is justified that the gap length \( l \) and width \( b \) are proportional to the module pitch \( n \) of the teeth in the mechanism:

\[ l = A_1 \cdot m \]  

(9)

\[ b = A_2 \cdot m \]  

(10)

The motors were examined using:
- the oil Total Azolla 46 (\( \nu = 40 \text{ cSt}, \rho = 873 \text{ kg/m}^3 \));
- 1 % emulsion HFA–E made on the basis of the Isosynth VX110BF concentrate (1% concentrate in water) (\( \nu = 0.853 \text{ cSt}, \rho = 996 \text{ kg/m}^3 \)).

RESULTS OF EXPERIMENTAL EXAMINATION

The characteristics of oil and emulsion leakage flows in flat motor gaps are shown in Fig. 6, while Fig. 7 presents the characteristics of leakage flow ratios in these gaps.

The results of the experimental examination have revealed that the ratio of the emulsion flow rate to the oil flow rate in these gaps can be described by the equation:

\[ \frac{Q_{LfgE}}{Q_{LfgO}} = E \cdot \Delta p_i^F \]  

(11)

where \( E \) is a constant and \( F \) is a coefficient. The values of \( E \) and \( F \) for the examined motors are given in Table 2.
order to calculate the constant E it was assumed that the
coefficient F should be constant. The value of F assumed
for further calculations was calculated as the average value
over the experimental data of all motors. On the basis of the
experimental data the constant E could be calculated for each
motor individually using the least square method (Table 2).

**Tab. 2. Values of constants E and F**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Motor</th>
<th>From experiment</th>
<th>Calculated</th>
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</thead>
<tbody>
<tr>
<td>E [-]</td>
<td>SM-0.4/20</td>
<td>41.566</td>
<td>39.941</td>
</tr>
<tr>
<td></td>
<td>SM-0.5/25</td>
<td>46.726</td>
<td>45.706</td>
</tr>
<tr>
<td></td>
<td>SM-0.6/25</td>
<td>31.5</td>
<td>32.118</td>
</tr>
<tr>
<td></td>
<td>SM-0.75/25</td>
<td>35.247</td>
<td>35.0</td>
</tr>
<tr>
<td>F [-]</td>
<td>SM-0.4/20</td>
<td>-0.218</td>
<td>-0.220</td>
</tr>
<tr>
<td></td>
<td>SM-0.5/25</td>
<td>-0.211</td>
<td>-0.200</td>
</tr>
<tr>
<td></td>
<td>SM-0.6/25</td>
<td>-0.140</td>
<td>-0.200</td>
</tr>
<tr>
<td></td>
<td>SM-0.75/25</td>
<td>-0.252</td>
<td>-0.200</td>
</tr>
<tr>
<td>E [MPa²]</td>
<td>SM-0.4/20</td>
<td>1.65</td>
<td>1.65</td>
</tr>
<tr>
<td></td>
<td>SM-0.5/25</td>
<td>1.65</td>
<td>1.65</td>
</tr>
<tr>
<td></td>
<td>SM-0.6/25</td>
<td>1.65</td>
<td>1.65</td>
</tr>
<tr>
<td></td>
<td>SM-0.75/25</td>
<td>1.65</td>
<td>1.65</td>
</tr>
</tbody>
</table>

**MATHEMATICAL MODEL**

Below are characterised two methods of modelling the
flow in flat gaps of the satellite motor working mechanism.
In both methods the following simplifying assumptions were
adopted:

− the viscosity of the medium in the gaps is constant. In
the real flow, the rise of the medium temperature in the
gaps is observed. For instance: at Δpᵢ = 25 MPa the oil
flow temperature rises by 12.5 °C, while the water flow
temperature by 6 °C. Consequently, the oil viscosity
decreases approximately by 22 % and the emulsion
viscosity by about 12 %;
− the gap walls are parallel to each other. In the loaded motor
the gap walls can be skewed, especially as a result of action
of the axial clearance compensation unit;
− the height of all satellites and the rotor is the same. However,
in the real motor the heights of these elements differ from
each other and, consequently, the heights of flat gaps are
also different;
− the gaps are situated only on one side of a satellite and the
rotor. In the real motor in operation, gaps can be observed
on both front sides of the rotor and the satellites.

**Method 1**

The pressure drop in the gap can be described by the
equation [11]:

\[
Δpᵢ = \frac{F}{β} \cdot \frac{1}{2} \cdot \frac{c²}{h},
\]

where:

\( c \) – velocity of medium flowing in the gap,
\( ρ \) – medium density.

In the literature, for the non-fully developed turbulent flow
a number of formulas can be found which describe the loss
coefficient \( λ \) as a function of the Reynolds number \( Re \). The
formula for loss coefficient which is suggested here for further
analyses has the form:

\[
λ = \frac{K}{Re^β}
\]

The flow velocity in the gap can be described using the
following formula:

\[
c = \frac{Q_{lfg}}{A₂ \cdot h}
\]

while the Reynolds number is [11]:

\[
Re = \frac{c \cdot h}{ν}
\]

where:

\( β \) – degree of laminarity of the flow (for the laminar flow \( β =
1 \), and for the turbulent flow \( β = 0 \), i.e.; \( β \in < 0;1 > \)
\( ν \) – kinetic viscosity of the liquid.

After placing Equations (13), (14) and (15) into Equation (12) and further transformations we arrive at:

\[
Q_{lfg} = \left( \frac{1}{ν} \right) \left( \frac{β}{2-β} \right) \left( \frac{1}{K} \cdot ρ \right) \cdot \left( \frac{1}{2-β} \right) \cdot Aₙ \cdot m \cdot \left( \frac{2}{A₂ \cdot m} \right) \cdot Δpᵢ
\]

When analysing the obtained results it was noticed that the
pressure increase is accompanied by the increasing level of
flow disturbance in the motor gaps, and this disturbance level
depends of the viscosity of the liquid. Consequently, the value
of the coefficient \( β \) will also depend on the viscosity of the
liquid and the pressure drop inside the motor. That means that
for a given \( Δp \); \( βₜ ≠ βₕ \). The subscripts O and E refer to the oil
and the emulsion, respectively.

The values of the coefficients \( K, Aₙ, A₂ \) and \( β \) can be
calculated from experimental data using the non-linear least
square method with the Levenberg-Marquardt algorithm. In this case we should assume that for oil flow in motor gaps $\beta_i \approx 1$. When the pressure drop in the motor increases, the height $h$ of the gaps decreases (the effect of action of the compensation unit) and the velocity of the medium flowing in these gaps changes. Therefore we can expect that $\beta_0 > 1$.

The relation between $\beta_i$ and $\beta_0$ can be calculated from the ratio of the emulsion and oil flows in the motor gaps. Hence, taking into account the relation (16), the ratio of the emulsion to oil flow rates is equal to:

$$Q_{E,E} = \frac{Q_{E,O}}{Q_{O,O}} = \frac{v_O}{v_E} \frac{\beta_O}{\beta_E} \left( \frac{\rho_O}{\rho_E} \right) \left( \frac{1}{\beta_O} \right) \left( \frac{1}{\beta_E} \right) \left( \frac{2-h}{A_2} \right)^m \Delta p_i \left( \frac{1}{2-\beta_E} \right) \left( \frac{1}{2-\beta_O} \right)$$

Comparing the formulas (11) and (17) gives:

$$E = \frac{v_O}{v_E} \frac{\beta_O}{\beta_E} \left( \frac{\rho_O}{\rho_E} \right) \left( \frac{1}{\beta_O} \right) \left( \frac{1}{\beta_E} \right) \left( \frac{2-h}{A_2} \right)^m \Delta p_i \left( \frac{1}{2-\beta_E} \right) \left( \frac{1}{2-\beta_O} \right)$$

Equation (11) and (17) give:

$$\beta_E = \frac{1}{2-\beta_O} - \frac{1}{2-\beta_0}$$

Consequently the coefficient $E$ depends on parameters of the liquid, geometry of the gaps, and the degrees of laminarity of the flows of oil and emulsion. At the same time the constant $F$ is only dependent on the degrees of laminarity of the emulsion and oil flows in flat gaps of the motor.

From Equation (19) we get:

$$F = \frac{1}{2-\beta_0} - \frac{1}{2-\beta_0}$$

while after taking into account Equations (19) and (20), Equation (18) takes the form:

$$E = \left( \frac{v_O}{v_E} \right) \left( \frac{\beta_O}{\beta_E} \right) \left( \frac{\rho_O}{\rho_E} \right) \left( \frac{1}{\beta_O} \right) \left( \frac{1}{\beta_E} \right) \left( \frac{2-h}{A_2} \right)^m \Delta p_i \left( \frac{1}{2-\beta_E} \right) \left( \frac{1}{2-\beta_O} \right)$$

The solution to this equation is:

$$\beta_O = \frac{2 \ln \left( E^{\frac{m}{2}} \frac{v_O}{v_E} \rho_O \rho_E \left( \frac{1}{2-\beta_E} \right) \left( \frac{1}{2-\beta_O} \right) \left( \frac{2-h}{A_2} \right)^m \Delta p_i \left( \frac{1}{2-\beta_E} \right) \left( \frac{1}{2-\beta_O} \right) \left( \frac{2-h}{A_2} \right)^m \Delta p_i \right)^F}{\ln \left( E^{\frac{m}{2}} \frac{v_O}{v_E} \rho_O \rho_E \left( \frac{1}{2-\beta_E} \right) \left( \frac{1}{2-\beta_O} \right) \left( \frac{2-h}{A_2} \right)^m \Delta p_i \left( \frac{1}{2-\beta_E} \right) \left( \frac{1}{2-\beta_O} \right) \left( \frac{2-h}{A_2} \right)^m \Delta p_i \right)}$$

Therefore both parameters $\beta_i$ and $\beta_0$ are functions of the gap height, which in turn depends on the pressure drop in the motor.

In order to find the relation between coefficients $K_0$ and $K_E$, simplifying assumptions have been adopted that $\beta_0 = 1$ and $h = h_0$. In that case from Equation (22) we get:

$$K_0 = K_E \left( \frac{F+1}{F+2} \right) \cdot \frac{p_E}{p_O} \cdot \frac{v_E}{v_O} \left( \frac{1}{2} \cdot \frac{m}{h_0^2} \cdot \frac{v_E^2}{v_O^2} \cdot \frac{p_E}{p_O} \cdot \frac{A_2}{A_2} \right)^F$$

The values obtained for the examined motors are given in Table 3, while Fig. 8 shows the characteristics of $\beta_i$ and $\beta_0$ as functions of $\Delta p_i$.

The calculated results presented in Table 3 indicate that the following simplified relation between the coefficients $K_0$ and $K_E$ exists:

$$K_0 = K_E \left( \frac{F+1}{F+2} \right) \cdot \frac{p_E}{p_O} \cdot \frac{v_E}{v_O} \left( \frac{1}{2} \cdot \frac{m}{h_0^2} \cdot \frac{v_E^2}{v_O^2} \cdot \frac{p_E}{p_O} \cdot \frac{A_2}{A_2} \right)^F$$

For the laminar flow of both the emulsion and the oil we have: $\beta_i = \beta_0 = 1$, while for the turbulent flow: $\beta_i = \beta_0 = 0.5$. After assuming $\beta_i = \beta_0 = \beta$ and comparing Equations (11) and (17) we can calculate $\Delta p_i$ for the laminar and turbulent flows of the both liquids:

$$E \cdot \Delta p_i^F = \left( \frac{v_O}{v_E} \right) \left( \frac{1}{2-\beta} \right) \left( \frac{K_0}{K_E} \rho_O \rho_E \left( \frac{1}{2-\beta} \right) \right)^F$$

---

**Tab. 3. Coefficient of model 1 at the assumption that $F = -0.2$**

<table>
<thead>
<tr>
<th>Coefficient/constant</th>
<th>Motor</th>
<th>SM-0.4/20</th>
<th>SM-0.5/25</th>
<th>SM-0.6/25</th>
<th>SM-0.75/25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$ [-]</td>
<td>---</td>
<td>0.580</td>
<td>0.125</td>
<td>0.145</td>
<td>0.17</td>
</tr>
<tr>
<td>$A_2$ [-]</td>
<td>---</td>
<td>0.396</td>
<td>0.0393</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>$K_0$ [m/(MPa’s)]</td>
<td>---</td>
<td>1.141</td>
<td>1.140</td>
<td>1.141</td>
<td></td>
</tr>
<tr>
<td>$K_1$ [mm/MPa]</td>
<td>---</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

---

**Fig. 9. Characteristics of oil and emulsion leakage flows in flat motor gaps (result of calculations)**

For the laminar flow of both the emulsion and the oil we have: $\beta_i = \beta_0 = 1$, while for the turbulent flow: $\beta_i = \beta_0 = 0.5$. After assuming $\beta_i = \beta_0 = \beta$ and comparing Equations (11) and (17) we can calculate $\Delta p_i$ for the laminar and turbulent flows of the both liquids:
For $\beta = 0$ the limiting pressure above which the turbulent flow of the both liquids takes place is:

$$\Delta p_{l,t} = \frac{F}{E} \cdot \left( \frac{\rho_e}{\rho_o} \right)^{0.5} \left( \frac{K_E}{K_O} \right)$$

(26)

The quantity $\Delta p_{l,t}$ takes values which are not reachable in technique. That means that the fully developed turbulent flow of oil will never occur.

For $\beta = 1$ the limiting pressure below which the laminar flow of the both liquids takes place is:

$$\Delta p_{l,l} = \frac{F}{E} \cdot \left( \frac{\rho_e}{\rho_o} \right) \left( \frac{K_E}{K_O} \right)$$

(27)

The values for $\Delta p_{l,l}$ are given in Table 4.

**Table 4.** $\Delta p_{l,t}$ values for the examined motors

<table>
<thead>
<tr>
<th>Motor:</th>
<th>SM-0.4/20</th>
<th>SM-0.5/25</th>
<th>SM-0.6/25</th>
<th>SM-0.75/25</th>
</tr>
</thead>
<tbody>
<tr>
<td>for $F$ – acc. to experiment</td>
<td>---</td>
<td>0.98</td>
<td>0.058</td>
<td>0.32</td>
</tr>
<tr>
<td>for $F = - 0.20$</td>
<td>0.87</td>
<td>0.15</td>
<td>0.23</td>
<td></td>
</tr>
</tbody>
</table>

**Method II**

This method assumes that:

a) in the region of cooperation of working mechanism teeth, where the gap length reaches the minimum, the flow is turbulent;

b) in the gaps with maximal length (satellite centre and rotor front) the flow is laminar;

c) the region of turbulent flow increases and the region of laminar flow decreases with the increasing pressure drop in the working mechanism:

$$b = X \cdot b_l + (1 - X) \cdot b_t$$

(28)

where:

$b_l$, $b_t$ – gap widths corresponding to the laminar and turbulent flow, respectively;

$X$ – gap width weight $b$.

The proposed definition of the coefficient $X$ is:

$$X = e^{-\left( \frac{Re}{C_1} \right)}$$

(29)

where $C_1$ is the weight coefficient and the Reynolds number is [11]:

$$Re = \frac{c \cdot h}{\nu}$$

(30)

The average flow velocity $c$ in the flat gaps of the working mechanism affects the flow rate $Q_{Lfg}$ and is not known. Therefore the following simplifying assumption is proposed:

$$c = C_2 \cdot \Delta p_l$$

(31)

where $C_2$ is the proportionality constant. Then:

$$X = e^{-\left( \frac{C_1}{C_2} \Delta p_l \right)}$$

(32)

where:

$$C = C_2 / C_1$$

(33)

The values of the constant $C$ should be different for the oil and the emulsion.

The laminar flow $Q_l$ in the gap can be modelled by the formula:

$$Q_l = X \cdot A \cdot \frac{1}{\rho} \cdot h^3 \cdot \Delta p_l$$

(34)

in which:

$$A = \frac{1}{12} \cdot \frac{b}{l}$$

(35)

and $b_l$ is the gap width in the motor area where the laminar flow is observed.

The turbulent flow $Q_t$ can be modelled by the formula:

$$Q_t = (1 - X) \cdot B \cdot m \cdot h \cdot \sqrt{\frac{2}{\rho} \cdot \Delta p_l}$$

(36)

In the above formula $B$ is the proportionality coefficient meeting the condition:

$$B \cdot m = B_1 \cdot b = B_1 \cdot A_2 \cdot m$$

(37)

In the discussed case the flow rate $Q_{Lfg}$ in the flat motor gaps can be expressed as the sum of the laminar component $Q_l$ and the turbulent component $Q_t$:

$$Q_{Lfg} = Q_l + Q_t$$

(38)

After relevant replacements, Equation (38) takes the form:

$$Q_{Lfg} = A \cdot e^{-C \left( \frac{h_R + h_S}{2} \cdot \Delta p_l \right)} \cdot \frac{1}{\rho} \cdot \left( \frac{h_R + h_S}{2} - D \cdot \Delta p_l \right)^3 \cdot \Delta p_l +$$

$$+ B \cdot m \cdot \left( 1 - e^{-C \left( \frac{h_R + h_S}{2} \cdot \Delta p_l \right)} \right) \cdot \left( \frac{h_R + h_S}{2} - D \cdot \Delta p_l \right) \cdot \sqrt{\frac{2}{\rho} \cdot \Delta p_l}$$

(39)

In the above formula the coefficients $A$ and $B$ and the constant $C$ are unknown. Their values for the examined motors were calculated from the experimental data $Q_{Lfg} = f(\Delta p_l)$ using the nonlinear least square method and the Levenberg–Marquardt algorithm. The values of the coefficients $A$ and $B$, and the constant $C$ are given in Table 5.

The characteristics of the oil and emulsion flow rates in flat gaps of working mechanisms of the examined motors, worked out using Equation (38) and the data from Table 2, are shown in Fig. 10.
DISCUSSION

The results of examination of the selected motors have proved that for a given pressure drop in flat gaps of the working mechanism of the motor, the flow rate of the 1 % HFA-E emulsion is at least from ten to twenty times larger than the oil flow rate. The dynamic viscosity of the oil is more than forty times larger than that of the 1 % HFA-E emulsion, and in this context it would be reasonable to expect that the flow rate of the emulsion is also more than forty times larger than the oil flow rate. However, this is true only for a narrow range of motor pressure drop. The present analysis has revealed that the pressure limit \( \Delta p_{\text{LT}} \) above which disturbances begin to appear in the emulsion flow is not the same for all motors. In the motors with typical clearances (up to 5 \( \mu m \)) this pressure can even reach 1 MPa, while in motors with very small working mechanism module pitches and large axial clearances (see the SM-0.4/20 motor) the flow disturbance is already observed as early as for \( \Delta p_{\text{LT}} > 0 \).

At large motor pressure drops, the ratio of the emulsion flow rate to the oil flow rate ranges within 15 ÷ 25.

Analysing the results obtained using method I makes it possible to formulate a conclusion that the decreasing height of the working mechanism gaps, being the result of the action of the clearance compensation unit, leads to the decrease of the coefficients \( \beta_0 \) and \( \beta_E \) (Fig. 8). That means that increasing \( \Delta p \) in the motor results in the increase of the flow velocity and intensification of the level of flow disturbances. Moreover, the value of the coefficient \( K_0 \) for oil is larger than the value of \( K_E \) for emulsion, and their ratio is equal to the density ratio between emulsion and oil. Therefore the flow resistance coefficients for oil and emulsion can be defined in the following way:

\[
\lambda_0 = \frac{1}{\text{Re}^0} \quad \text{and} \quad \lambda_E = \frac{1}{\text{Re}^E}
\]

Having known the value of the coefficient \( \beta \) we can calculate the proportion \( S_{t-I} \) of the turbulent component in the flow rate \( Q_{t-I} \) in the motor gaps. This component, expressed in %, is equal to:

\[
S_{t-I} = (1 - \beta) \cdot 100\%
\]

The percent proportion \( S_{t-II} \) of the turbulent component of the flow rate in the gaps calculated using method II (formula (38)) is equal to:

\[
S_{t-II} = \left( \frac{Q_t}{Q_{\text{t-II}}} \right) \cdot 100\%
\]

The values of \( S_{t-I} \) and \( S_{t-II} \) are given in Table 6.

It is noticeable that smallest differences between the values of \( S_{t-I} \) and \( S_{t-II} \) are recorded for large pressure drops in the gaps and for low-viscosity liquids.

Based on the values of the coefficients A and B given in Table 5 we can conclude that:

- the largest turbulence area in the gaps is observed in motors with the highest gap height-to-length ratio – here it is the motor SM-0.4/20. That is why model I coefficients could not be calculated for this motor;
- for all motors we can assume:

\[
\frac{A_0}{A_E} \approx 2, \quad \frac{B_E}{B_0} \approx 7
\]
In order to compare the accuracy of the method I and II with the experiment, the percent deviations of the results of $Q_{Lfg}$ calculations were shown in Table 7.

Comparing the result collected in Table 7, we can see that the smallest $Q_{Lfg}$ deviations from the experiment have been obtained using method I.

**SUMMARY**

The article presents the methodology and results of experimental investigations of the flow rate in flat gaps of satellite motor working mechanisms. These gaps are characteristic for variable length and width. The results of the experiments have made it possible to assume that the flow in these gaps is turbulent but not fully developed. In this context two methods were proposed using which the flow can be calculated for liquids revealing large viscosity differences. The results of these calculations have proved that for the nominal pressure drop in the motor with nominal gap heights the turbulent flow component in the gap can amount to 3 % for oil, and as much as to 30 % for the emulsion.

In the future, the here presented methods to describe the liquid flow in flat gaps of the motor working mechanism are planned to be verified using the CFD method. What is also planned is the numerical calculation of the flow of liquid in elementary gaps (of rectangular or toroidal shape, etc.) a, for different-viscosity liquids, in particular for water.

Methods I and II can be used for working out models of volumetric losses in the motor. For this purpose the flow in the gaps of motor distribution system, as well as the flow convected in the inter-teeth spaces of the working mechanism, the flow caused by cyclic deformation of working chambers, and the flow caused by compressibility of the liquid, are to be modelled mathematically. These issues will be the subject of the next publications.

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**Table 7. Difference between QLfg, in %, obtained experimentally and calculated using method I and II**

<table>
<thead>
<tr>
<th>Liquid</th>
<th>ΔpLfg [MPa]</th>
<th>Method</th>
<th>Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>SM-0.4/20</td>
</tr>
<tr>
<td>Oil</td>
<td></td>
<td>I</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>1.6</td>
</tr>
<tr>
<td>Emulsion</td>
<td></td>
<td>I</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>15.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Bead-on-plate welding on S235JR steel by underwater local dry chamber process

Grzegorz Rogalski, Ph. D., Jerzy Łabanowski, Assoc. Prof., Dariusz Fydrych, Ph. D., Jacek Tomków, M. Sc., Gdańsk University of Technology, Poland

ABSTRACT

The article presents the results of the effect of parameters of underwater local dry chamber welding on the properties of padding welds. The effect of heat input and the type of shielding gas on the structure and hardness of welds was established. The functions for estimating the maximum hardness of the heat affected zone have been also elaborated.

Keywords: maximum hardness of the heat affected zone; welding under water; dry chamber process

INTRODUCTION

Underwater welding is most often applied to repair – maintenance work but also in joining new elements, e.g. the welding of corrosion protection (cathodes) or quay elements [1, 2]. Wet welding technique is usually applied to underwater welding [3 ÷ 11]. It is characterized by a relatively low welding cost, easiness of manipulation of welding torch, but it also has certain disadvantages. Among the most important can be numbered the following: a greater cooling down rate of welded joint, greater amount of diffusing hydrogen as well as limitation of water depth for underwater work to about 50 m. Quality level of underwater welded joints depends to a large extent on diver – welder skill, but to reach it a special training is required, and even that does not guaranty an appropriate quality of welded joints. Dry welding techniques which make it possible to obtain joints of identical features compared with those made in air atmosphere, have been elaborated, but such methods are very expensive [12]. It is connected with the building of special working chambers coupled with main units. Implementation of a local dry chamber which combines some features of wet and dry welding, has appeared to be an alternative to the both ways of underwater welding [13 ÷ 17]. In the method in question a diver-welder is located under water and welding process is carried out within a space isolated from water by means of special permanent or moveable chambers. The most simple and cheap solution is to apply a local dry chamber directly fixed with welding torch. Cost of the methods is much lower than that of dry welding, and features of welded joints produced this way are close to those of the joints made in dry conditions [15, 16]. Fig. 1 shows the principle of underwater welding by using the method of local dry chamber.

In underwater welding special attention should be paid to possible cold fracture forming. Water environment is a source of diffusing hydrogen, and short cooling down times in the temperature range of 800 ÷ 500 °C (t8/5) contribute to the forming of hardening structures within welding joint. Therefore it is so important to be capable of predicting structures to be formed during welding. It can be realized, a. o., on the basis of hardness analysis for particular areas of welded joint. To determine influence of a type of shielding gas on hardness of padding welds is very important not only from metallurgical point of view but also for economical
reasons. The shielding gas \( \mathrm{CO}_2 \) is cheap, and, additionally, the welding with the use of MAG (135) method in air environment is considered a low-hydrogen process, that is of a great importance in underwater conditions. Shielding gas fulfils double role during welding with the use of local dry chamber. It protects liquid metal pool and removes water from welding area. Therefore in this case shielding gas flow rate is greater than that during conventional welding process with the use of the MAG (135) method in air. It depends on a welding water depth and local chamber size. At the water depth of 1 ÷ 2 m the gas flow rate is comprised within the range of 30 ÷ 40 l/min, and at greater depths it can exceed 100 l/min. Therefore during welding at greater depths cost of shielding gas consumption can be significant.

**EXPERIMENTAL TESTS**

The experimental tests were aimed at determining influence of amount of heat input (welding linear energy) as well as kind of shielding gas on hardness and structure of padding welds made underwater. The tests were focused on determining analytical functions making it possible to predict maximum hardness of weld produced underwater by using the local dry chamber method.

**CONDITIONS FOR REALIZATION OF EXPERIMENTAL TESTS**

The tests were performed on padding welds. On the basis of literature analysis [17] it has been concluded that that differences between values of the cooling time \( t_{8/5} \) for padding welds and welds made in a „V“- groove butt joint are rather low (Fig. 2). Hence it can be expected that structures obtained within padding weld area will be similar to those formed in the butt joint with butt weld.

The welding water depth was assumed on the level of 0.5 m. Water salinity was equal to the average salinity of seas and oceans (13 ‰). Amount of heat input was assumed on the level which makes it possible to produce root runs and filling ones. Values of the electric current parameters and remaining crucial variables are presented in Tab. 1.

**MATERIALS USED FOR THE TESTS**

The padding welds were laid on the plates of 20 mm in thickness, made of S235JR non-alloy steel (acc. the material group 1.1 of PN CR ISO 15608 standard). The steel is characterized by a very good weldability and does not produce any troubles during welding in air. Hardness of its ferritic-pearlitic structure does not exceed 180 HV10. To produce the samples a welding wire of 2 mm diameter, marked G 38 2 C G3Si1 (acc. PN-EN ISO 14341-A), was used. Chemical composition of the steel and additional welding material is given in Tab. 2.

The welding was performed with the use of the 135 (MAG) method in atmosphere of the three commonly available shielding gases:
1. \( \mathrm{CO}_2 \), acc. PN-EN ISO 14175 – C1 (100% \( \mathrm{CO}_2 \)),
2. MIX18, acc. PN-EN ISO 14175 – M21 (18% \( \mathrm{CO}_2 + 82\% \text{Ar} \)),
3. H5, acc. PN-EN ISO 14175 – R1 (5% \( \mathrm{H}_2 + 95\% \text{Ar} \)).

**Fig. 2. Influence of joint type on the value of cooling time \( t_{8/5} \) [17]**

**Tab. 1. Welding parameters of samples**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Shielding gas acc. to PN-EN ISO 14175</th>
<th>Voltage ( U ) [V]</th>
<th>Welding current ( I ) [A]</th>
<th>Welding speed ( V_{wp} ) [m/min]</th>
<th>Gas flow rate ( W_g ) [l/min]</th>
<th>Heat input ( E_L ) [kJ/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C I</td>
<td>C1</td>
<td>30</td>
<td>155</td>
<td>0.305</td>
<td>35</td>
<td>0.91</td>
</tr>
<tr>
<td>C II</td>
<td>C1</td>
<td>38</td>
<td>170</td>
<td>0.305</td>
<td>35</td>
<td>1.27</td>
</tr>
<tr>
<td>C III</td>
<td>C1</td>
<td>43</td>
<td>205</td>
<td>0.305</td>
<td>35</td>
<td>1.73</td>
</tr>
<tr>
<td>M I</td>
<td>M21</td>
<td>30.3</td>
<td>152</td>
<td>0.305</td>
<td>35</td>
<td>0.90</td>
</tr>
<tr>
<td>M II</td>
<td>M21</td>
<td>30.5</td>
<td>236</td>
<td>0.305</td>
<td>35</td>
<td>1.41</td>
</tr>
<tr>
<td>M III</td>
<td>M21</td>
<td>30.8</td>
<td>236</td>
<td>0.245</td>
<td>35</td>
<td>1.78</td>
</tr>
<tr>
<td>R I</td>
<td>R1</td>
<td>30.8</td>
<td>132</td>
<td>0.245</td>
<td>35</td>
<td>0.99</td>
</tr>
<tr>
<td>R II</td>
<td>R1</td>
<td>31.8</td>
<td>204</td>
<td>0.305</td>
<td>35</td>
<td>1.27</td>
</tr>
<tr>
<td>R III</td>
<td>R1</td>
<td>40.3</td>
<td>216</td>
<td>0.305</td>
<td>35</td>
<td>1.71</td>
</tr>
</tbody>
</table>

**Tab. 2. Chemical composition of materials used in the tests**

<table>
<thead>
<tr>
<th>Material</th>
<th>Chemical composition [weight %]</th>
</tr>
</thead>
<tbody>
<tr>
<td>S235JR steel</td>
<td>C 0.17 ÷ 0.20 Mn max.1.40 Si -- P max.0.045 S max.0.045 N max.0.009</td>
</tr>
<tr>
<td>Welding wire G 38 2 C G3Si1</td>
<td>0.10 0.88 0.26 0.013 0.010 --</td>
</tr>
</tbody>
</table>
TEST STAND FOR UNDERWATER WELDING AND CUTTING AT SHALLOW WATER DEPTHS

The padding welding was carried out on the test stand for underwater welding and cutting at shallow water depths (up to 1 m). It makes welding and padding welding in various positions as well as monitoring the processes, possible. It is equipped with ESAB ARISTO 400 welding plant (power source) which allows to record electric current parameters of the process. Component elements of the test stand are presented in Fig. 3.

RESULTS OF THE TESTS AND THEIR ANALYSIS

Hardness measurements were performed by means of Vickers (HV10) method on padding weld cross-sections. The test was carried out in compliance with PN-EN 1043-1 standard. The measurements were made in the points 2 mm distant from sample’s edge, Fig. 4. The distance between imprint centres was not smaller than L=1mm. The test results are presented in Fig. 5 through 7.

When analyzing the hardness distributions over cross-sections of the padding welds in question it is observed that the greatest hardening in HAZ (abt. 400 HV10) was obtained for the samples marked MI and MII, welded in atmosphere of the shielding gas M21 (18% CO₂ + 82% Ar). Large hardness values in HAZ were also noted for the samples welded in atmosphere of the shielding gas C1 (100% CO₂), on the contrary the samples welded under the shielding gas R1 (5% H₂ + 95% Ar) do not show any large hardening in HAZ, and differences in hardness values of their padding weld material and HAZ are rather low (Tab. 3).

If the hardness recommendations contained in PN-EN ISO 15614-1 standard, are taken to be an acceptance criterium (i.e. 380 HV10 – without heat treatment) then the samples M I and M II do not satisfy the standard’s requirements (Tab. 3). The large hardness areas may show hardening structure at fusion line (for S235JR steel - bainitic structure). In underwater welding conditions it can lead to forming cold fractures.
Increase of the heat input transferred during welding within the range of 0.9 ÷ 1.7 kJ/mm, resulted in lowering maximum values of hardness in HAZ of padding welds, however no linear relation was observed. The maximum hardness values of the samples welded under the shielding gas M21 and C1 did not significantly differ from those obtained for the welding heat input of 0.9 and 1.3 kJ/mm. Only the application of the energy of abt. 1.7 kJ/mm in value resulted in HVmax decreasing in HAZ. For the samples prepared under the shielding gas R1 the influence of the welding heat input on the maximum hardness values in HAZ was rather slight.

It is characteristic that the padding weld metal hardness measured in the middle of weld breadth amounted to about 240 ÷ 250 HV for the samples welded under the argon-based shielding gases – M21 and R1, but the hardness of padding weld prepared under the shielding gas CO2 (C1gas) was lower and equal to 210 ÷ 230 HV.

Application of different shielding gases affects geometry of obtained padding welds, i.e. their shape and fusion depth. Pictures of cross-sections of the obtained padding welds were presented in Fig. 8.

The increased hardness in the padding weld HAZ indicates that the building of hardening structures in this area and applied welding conditions, is possible. In order to reveal microstructures of padding welds, microscopic metallographic examinations were performed on the samples comprising padding weld cross-sections. The examinations were carried out with the use of NEOPHOT 32 optical microscope. In Fig. 9 through 11 are presented the observed microstructures of some selected samples.

The metallographic examinations demonstrated that the application of the shielding gas M21, at the welding heat input of 0.9 kJ/mm and 1.3 kJ/mm, resulted in forming the bainitic structure in HAZ, Fig. 9. In the remaining padding welds made with even lower heat input values, presence of any hardening structures in HAZ was not observed.

### Tab. 3. Maximum HAZ hardness of the padding welds

<table>
<thead>
<tr>
<th>Weld</th>
<th>HV10 max HAZleft</th>
<th>Weld</th>
<th>HV10 max HAZright</th>
</tr>
</thead>
<tbody>
<tr>
<td>C I</td>
<td>333</td>
<td>-230</td>
<td>285</td>
</tr>
<tr>
<td>C II</td>
<td>309</td>
<td>-210</td>
<td>290</td>
</tr>
<tr>
<td>C III</td>
<td>279</td>
<td>-200</td>
<td>281</td>
</tr>
<tr>
<td>M I</td>
<td>394</td>
<td>-250</td>
<td>394</td>
</tr>
<tr>
<td>M II</td>
<td>401</td>
<td>-260</td>
<td>398</td>
</tr>
<tr>
<td>M III</td>
<td>360</td>
<td>-250</td>
<td>351</td>
</tr>
<tr>
<td>R I</td>
<td>268</td>
<td>-260</td>
<td>272</td>
</tr>
<tr>
<td>R II</td>
<td>254</td>
<td>-240</td>
<td>254</td>
</tr>
<tr>
<td>R III</td>
<td>264</td>
<td>-240</td>
<td>258</td>
</tr>
</tbody>
</table>

![Fig 6. Distributions of the hardness HV10 on the cross-section of padding welds: a) M I, b) M II, c) M III](image)

![Fig 7. Distributions of the hardness HV10 on the cross-section of padding welds: a) R I, b) R II, c) R III](image)
In conditions of underwater welding with the use of the local dry chamber method a greater rate of heat transfer from HAZ area than in air conditions, should be expected. Apart from interaction of surrounding water, intensive shielding gas flow is the other factor which affects cooling rate of the joint. The gas is applied not only to shield electric arc but also to remove all amount of water out of chamber volume. Heat transfer intensity depends not only on flow rate of the gas but also on its physical properties.

From the made observations and measurements it results that at the same welding conditions (plate thickness, immersion depth, linear energy, gas flow rate) as well as application of different shielding gases significant differences occur in maximum hardness values in HAZ of padding welds. The differences may be attributed to different values of thermal conductance coefficient of the applied shielding gases, gas influence on forming electric arc and its properties as well as chemical reactions running in high temperature of electric arc in presence of a large amount of water vapour.

The application of the mixture of argon and hydrogen (R1), a gas of a high thermal conductance, resulted in forming non-concentrated arc which produces a wide padding weld of a low fusion depth (Fig. 8b). However the high thermal conductance of hydrogen did not result in increasing cooling-down intensity of padding weld and hardness in HAZ. The application of M21 mixture composed of argon and CO2, i.e. gases of low thermal conductance, gave a concentrated electric arc resulting in a characteristic bell form of the padding weld with deep fusion depth in the middle part (Fig. 8c). Such effect of the electric arc, at the intensive water-cooling of plate, resulted in the forming of hardening structures in HAZ and a significant rise of hardness.

Welding under the shielding gas CO2 resulted in the forming of the so-called “hot arc” which produces a wide padding weld of a deep fusion depth (Fig. 8a). Reactions of CO dissociation and CO2 recombination can cause phenomenon of increasing welding energy as well as burning-up alloying elements from steel to occur. To the last phenomenon can be associated decreasing the hardness in padding weld metal (Mn burning – up).

As observed, it is hard to unambiguously describe a way in which particular shielding gases affect heating and cooling intensity of HAZ of padding welds though the effects have been determined unambiguously. Moreover one should be always conscious of possible taking place of an untightness in local dry chamber and hence presence of water in welding area. Presence of water causes gas bladders close to padding weld face, and pores, to form.
The gas bladders initiate forming microfractures, which is presented in Fig. 12 and 13. Such defects lead to disqualification of a joint and its removal from operation.

**Fig. 12.** The face of the R III weld. Visible great number of pores

**Rys. 13.** The microstructure of R III sample. Gas bladder with apparent crack initiation sites

**ELABORATION OF FORMULAE FOR DETERMINING THE MAXIMUM HARDNESS IN HAZ OF THE JOINTS**

The performed tests made it possible to elaborate analytical formulae for estimating the maximum hardness in HAZ of the joints for the applied types of shielding gas as well as welding heat input values, Tab. 4. The formulae obtained from the tests, are presented in Fig. 14. As can be observed, trend lines of the diagram show an only slight decrease of hardness along with the increasing of welding heat input. This is in compliance with expectations as well as experience gained from welding in air atmosphere. However it is not possible to conclude whether it results from the effect of welding heat input itself, shielding gas type or - may be - an interaction of the both factors.

**Table 4.** The analytical functions for estimating the maximum hardness in HAZ, depending on the heat input and the type of shielding gas

<table>
<thead>
<tr>
<th>Shielding gas</th>
<th>Equation</th>
<th>Heat input [kJ/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M21</td>
<td>HV10&lt;sub&gt;max&lt;/sub&gt; = 35.356 · E&lt;sub&gt;L&lt;/sub&gt; + 433.41</td>
<td>0.75 - 1.2</td>
</tr>
<tr>
<td>C1</td>
<td>HV10&lt;sub&gt;max&lt;/sub&gt; = 25.014 · E&lt;sub&gt;L&lt;/sub&gt; + 340.35</td>
<td>1.2 - 1.8</td>
</tr>
<tr>
<td>R1</td>
<td>HV10&lt;sub&gt;max&lt;/sub&gt; = -8.3907 · E&lt;sub&gt;L&lt;/sub&gt; + 274.81</td>
<td>1.8 - 1.9</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

1. The maximum hardness in HAZ of the joints welded underwater with the use of the local dry chamber method depends on a type of applied shielding gas.
2. The application of the shielding gas M21 (18% CO<sub>2</sub> + 82% Ar) at the amount of heat input from the range of 0.9 ÷ 1.3 kJ/mm, resulted in an excessive rise of hardness within HAZ. The observed values of the hardness reaching 400HV suggest that a hardening structure has been formed, as confirmed by metallographic examinations.
3. The application of the shielding gas C1 (100% CO<sub>2</sub>) as well as R1 (5% H<sub>2</sub> + 95% Ar) at the heat input from the range of 0.9 ÷ 1.7kJ/mm, does not cause any excessive rise of hardness within HAZ of the joints.
4. The performed investigations made it possible to elaborate the analytical formulæ for determining the maximum hardness value within HAZ in case of underwater welding with the use of the local dry chamber method.
5. In order to take into account economic aspects as well as limit forming the faults like gas bladders and pores, the shielding gas C1 is recommended for underwater welding with the use of the local dry chamber method.

**BIBLIOGRAPHY**


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Introduction

One of the main tasks of the Navy is to track submarines (Anti-Submarine Warfare - ASW) and search for and destroy naval mines (Mine Counter-Measure - MCM) [7]. This is done by specialised ships equipped with sonar systems. Submarines are tracked by using long-range low frequency sonars. ASW tasks are also performed by helicopters equipped with dipping sonar and sonobuoy systems [7]. Naval mines are searched for and destroyed by minesweepers and destroyers using high resolution short range sonar [7]. With the rapid advancement of electronic technologies and digital signal processing methods, electronic systems, including sonars which are a key element of ASW and MCM ships, become obsolete very quickly. In the late 1990s a team of researchers of the Department of Marine Electronics Systems, Faculty of Electronics, Telecommunications and Informatics, Gdansk University of Technology, began work on modernizing existing sonar systems for the Polish Navy. As part of the effort, a methodology of sonar modernization was implemented involving a complete replacement of existing electronic components with newly designed ones by using bespoke systems and methods of digital signal processing. Large and expensive systems of ultrasound transducers and their dipping and stabilisation systems underwent necessary repairs but were otherwise left unchanged. As a result, between 2001 and 2014 the Gdansk University of Technology helped to modernize 30 sonars of different types.

Application of New Technologies

Modernized at the Gdansk University of Technology, the sonar systems operated by the Polish Navy were originally built in the 1970s and 1980s. The majority were Soviet designs built entirely in analogue technology, even with some of the transmitting equipment using high-power electron tubes. Following Poland’s accession to NATO, the Polish Navy received two US Oliver Hazard Perry class frigates, built in 1978 and 1980. Even though the frigates’ sonars were made in digital technology, it was the technology of the 1970s with sonars started by reading perforated tape and with signal processing computers for towed array sonar occupying entire fairly large cabins.

The main objective of the modernization was to design, build and replace all electronic systems and use modern digital systems in place of analogue or obsolete digital technology. Modernized at the Gdansk University of Technology, the sonar systems operated by the Polish Navy were originally built in the 1970s and 1980s. The majority were Soviet designs built entirely in analogue technology, even with some of the transmitting equipment using high-power electron tubes. Following Poland’s accession to NATO, the Polish Navy received two US Oliver Hazard Perry class frigates, built in 1978 and 1980. Even though the frigates’ sonars were made in digital technology, it was the technology of the 1970s with sonars started by reading perforated tape and with signal processing computers for towed array sonar occupying entire fairly large cabins.

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DIGITAL SIGNAL PROCESSING IN MODERNIZED SONARS

The world’s first sonar using digital signal processing (DSP) was developed in 1960 [47]. Since then digital signal processing methods have been continuously improved to keep up with the demands of the new developments [5, 6, 10, 15, 16, 41, 54, 63]. Equipped with digital signal processing, sonars are now capable of delivering a wide range of new functions which were not available or difficult to achieve in the analogue technology or early digital technology, respectively. This applies in particular to the generation of sounding signals, beam-forming, pulse compression, filtration, detection and imaging of echo signals. The widespread use of sonars with multi-element ultrasonic transducers raises the bar for DSP processors in sonars even more than for DSP processors used in radar.

The modernization conducted at the Gdansk University of Technology involved not only the design and construction of entirely new systems using cutting-edge electronic technology, but also the use of broadband soundings signals and modern methods of digital signal processing. Although a number of the methods can be found in the literature, some of them had to be adapted to the specific parameters of the equipment. Part of the modernization at the Gdansk University of Technology included the development of new signal processing algorithms which are extensively described in many publications [11, 13, 20, 21, 22, 25, 26, 29, 30, 45, 48, 49, 50, 51, 52, 53, 54].

The design of modern sonar systems requires transducer equipment to generate broadband sounding signals and receiver equipment to use sophisticated methods of digital echo signal processing.

Generating sounding signals by using DDS

Direct Digital Synthesis (DDS) [3, 4] is a modern digital method for generating arbitrary signals. A DDS integrated circuit includes a control microprocessor, quartz frequency generator, programmable counter, Digital to Analogue Converter (DAC) and output analogue filter. When used in sonar transmitters, DDS offers new unlimited potential for sounding signal generation, starting from the simple “ping” type signals, through “chirp” signals with linear (LFM) or hyperbolic (HFM) frequency modulation to any combination of the two.

The method was applied in all of the sonars modernized at the Gdansk University of Technology for generating new broadband sounding signals.

Multi-beam systems, which also use Rotational Directional Transmission (RDT) at the transmitting end, have to generate a number of simultaneous signals with phase shifts changing over time. The modernized MG-89DSP sonar uses multi-channel systems with multiple DDS systems. Thanks to the features of DDS, the shape of sounding pulses can now be optimized and helps to minimize the level of sounding signal side lobes, an effect which traditional technologies cannot possibly achieve [51].

Analogue - to - digital conversion in sonar systems

Once received by a multi-element ultrasonic transducer, echo signals of the modernized sonars are passed on to a multi-channel receiver for pre-processing. Signal pre-processing is conducted in the input analogue part of the receiver to ensure filtration and gain and to reduce the dynamics by applying time variable gain (TVG) [31]. Following this, echo signals undergo analogue - to - digital conversion in the multi-channel A/D converter to be then sent to specialized DSP processors. The process of analogue - to - digital conversion conducted in sonar systems must meet a number of conditions. Because the signals will be processed subsequent to the sampling for the purpose of time and space filtration, all channel sampling must be done synchronously in the same moments of time. To ensure the accurate timing of sampling, a number of independent converters are used rather than a single converter with input multiplexer, a common solution in other applications [46, 57]. The accuracy of the process depends on the input signal staying constant throughout the conversion time. The traditional approach is to use sample-hold systems but the more recent solutions use charge redistribution successive approximation analogue - to - digital converters. The converters are based on the Switched-Capacitor Circuits technology. The sample-hold process is at the core of their operation [12, 24].
Quadrature sampling

When we sample narrow-band signals with the frequency band B centred around the mid-channel frequency \( f_0 \), we can significantly reduce the amount of data collected by using quadrature sampling, also known in the literature as second-order sampling or IQ sampling (In-phase and Quadrature-phase Sampling) [2, 8]. The quadrature sampling involves collecting pairs of signal samples which are 1/4 of mid-channel frequency period apart. In practice the sampling frequency is four times the mid-channel frequency \( f_0 \), and the time lapse between pairs of samples which are selected is not more than 1/B. The first sample is treated as cophasal and the second as quadrature. Sequences of cophasal and quadrature samples are made. Because the spectra of cophasal and quadrature samples are within the sub-detection band \(-B/2 \) to \(+B/2\), the amount of data to be processed is of cophasal and quadrature samples are identical and the complex spectra of the sequences are conjugate.

For broadband signals with their spectrum centred around the mid-channel frequency \( f_0 \), and for narrowband signals with significant Doppler deviation, quadrature sampling introduces computational errors [1, 62]. Despite this, they can still be used; however, the results should be verified first.

Quadrature sampling is commonly used in modernized sonars. The application of broadband sounding signals meant that each narrowband approximation had to be analyzed for its effect on the accuracy of the results [52, 53]. That was the reason why traditional first-order sampling consistent with Nyquist theorem [45] was used in the modernization only in the case of broadband signal processing in passive SQR-19 sonar.

Digital Signal Processors

Digital signal processing involves a high number of mathematical operations on samples of the input signal. Digital Signal Processors are specialized processors whose architecture has been optimized to support fast computations which are typical in digital signal processing [23, 61]. Their main features include the so-called Harvard architecture with separate programme memory and data, pipelined execution of instructions and equipment implementation of the most typical signal processing operations such as FFT, FIR and IIR type filtrations and correlation.

Recent years have seen the emergence of new technologies which are in competition with specialized DSP processors. The development of General Purpose Processors (GPPs), very fast GPPs (as an example Intel, Core i7 type) can successfully perform digital signal processing in a number of applications. Their prices and availability make them a real alternative to specialized DSPs. This is called a Commercial Off-the-Shelf (COTS) or COTS GPPs solution [40].

Programmable FPGA (Field Programmable Gate Arrays) matrices are another growing possibility [40]. Their advantage is that they help with equipment miniaturization and use less power. The downside is that they are complicated to programme by using VHDL language (Very-high-speed integrated circuits Hardware Description Language). It is more difficult and takes more time than C or C++ programming, a language that can be used for programming classic DSPs. It is for these reasons that FPGAs are recommended for digital signal processing where size and power matter and in the case of larger scale manufacture.

For the above mentioned reasons, when modernizing the sonars, COTS GPPs processors were used as long as their computational power was sufficient. An example, the modernization of the helicopter on-board OKA sonar and the low frequency passive sonar with towed array SQR-19, involved the use of Industrial Computers, based on generally available processors.

In the case of modernizing active multi-beam sonars operating at higher frequencies, the Department of Marine Electronics Systems developed a multi-processor module based on DSPs produced by Texas Instruments TMS320C6713B. The module’s architecture was optimized to ensure that it meets sonar signal processing requirements and can multiply the number of modules if needed to increase computational power [46, 57]. The module is easily programmable in the language C++ (alternatively to programming in the assembler) allowing fast implementation of algorithms pre-tested in the MATLAB environment [54].

SONAR DIGITAL SIGNAL PROCESSING ALGORITHMS

The basic tasks of digital signal processing in sonar systems include spatial filtering, also known as beam-forming, spectral analysis, matched filtration and correlation analysis [5, 6, 10, 15, 16, 41, 54, 63]. The next sub-sections are concerned with how these tasks are completed in the modernized sonars. There are different ways to implement spatial filtering algorithms, depending on array shape and the frequency and bandwidth of the signals.

FFT beam-forming for sonar with linear array

The array of the MG-89 sonar is a typical linear multi-element ultrasonic transducer with its elements spaced every half wavelength. The beam-forming applied for modernizing this sonar is the digital version of the phase beam-former, operating on complex samples produced as a result of quadrature sampling. Fig. 2 illustrates the principle of generating a single beam deflected from the acoustic axis by a given angle. The individual sections of the acoustic array receive echo signals whose phase shifts depend on the wave’s angle of incidence. The direction of the beam pattern maximum is changed by changing the phase of signals received by the particular array elements. If the phase shifts are selected so as to ensure that signals at phase shifters’ outputs generated by a wave incident from a specific direction have identical phases, the signal amplitudes at adder output will be maximum. As a result, the beam pattern will turn by a given angle versus the array’s acoustic axis. If a number of deflected beams are to be generated, these operations should be carried out for each beam with the right phase shifts that match the beam’s desired angle of deflection. Generating 60 deflected beams requires 60 \( \times 360 = 2160 \) phase shifters. We can halve this number by using the properties of phase shift symmetry.

The shifts of echo signal phases in the modernized sonar MG-89D are made digitally [46]. A sinusoidal signal represented by its quadrature samples has its phase shifted by changing the proportions between sine and cosine samples. The samples are multiplied by ratios whose value is equal to the sine and cosine from the desired phase shift. Next, the results of the multiplication are added which yields sine and cosine samples again but this time with a changed phase. Following this transformation, the sine and cosine samples are added. The result is one sine and one cosine sample in each sampling cycle.

The root of the sum of squares of these samples is proportional to the value of the relevant beam pattern and for the given wave incidence angle.
DSP operations can be significantly accelerated by applying the FFT algorithm to spatial frequencies [5, 6, 15, 54]. Because the application of the FFT algorithm involves a linear distribution of phase shifts, the result is a non-linear distribution of beam deflection angles. This makes it appropriate for fairly small beam deflection angles, typically not exceeding ±30°. The application of the FFT algorithm goes hand in hand with the term of spatial frequencies, having to calculate the spectrum of the frequencies by using Fourier transformation. It can be seen that signal samples at array outputs have a sinusoidal shape of a specific frequency. The frequency depends on the wave incidence angle, hence the name-spatial frequency. If the signal samples undergo discrete Fourier transformation, the result will be a discrete spectrum. The lines of the spectrum are assigned to the individual angles of acoustic wave incidence on the array. To increase the number of beams and at the same time improve the sonar’s angular resolution, the sequence of samples is supplemented with zero samples and the total number of samples should be equal to \( n - \text{power of 2} \), which is a requirement of the FFT algorithm. This approach was used in the modernized MG-89DSP sonar’s algorithm to increase the number of samples up to 128. Thanks to this the desired number of 61 beams was achieved in a 60° angular sector. In addition, the algorithm includes amplitude weighing echo signals to reduce the level of side lobes. Selected beam patterns of the beam-former in question are shown in Fig. 3. Amplitude weighing is done by multiplying the values of sine and cosine samples by weight ratios. The resulting theoretical side lobe level of receiving characteristics generated in the digital beam-former is −18 dB. It is only in the case of a 30° deflected beam that you can see a −16 dB side lobe.

**Wideband beam-forming for passive sonar with towed linear array**

Because passive sonars receive and process broadband signals of shipping noise with frequency bands comprising several octaves [9, 17, 18, 19], spatial signal processing in the sonars is much more complex than described in the previous section. The SQR-19PG sonar with towed linear array is an example. The band of frequencies received comprises more than 7 octaves ranging from 10 Hz to 1400 Hz, the linear array is 192 m long and the observation sector is 360°.

Passive sonar determines the orientation of incoming acoustic wave by using broadband beam-forming algorithms or algorithms for spatial spectrum estimation. In practice both methods are used, the classical beam-forming as the basic mode of observation and spatial spectrum estimation for a more precise ranging.

Broadband beam-formers may be implemented in the domain of time or frequency. The idea of a broadband beam-former in the time domain is simpler. It directly implements signal delays to compensate for the difference between the paths of a wave coming to successive array elements from a specific direction (Fig. 2). To determine signal delays in the required resolution the number of signal samples is multiplied by using interpolation filtering. For this reason beam-formers operating in the time domain are called interpolation beam-formers. Because their principal computational effort is the consequence of interpolation, such beam-formers require a big amount of operating memory.

Beam-formers implemented in the frequency domain, on the other hand, have to have separate phase compensation for each spectral line of the received signals. As a result, when processing first begins, the signals are processed into spectral form by using FFT algorithms, and then each of the spectral lines is treated with narrowband phase beam-forming algorithms just as those described in the previous section. While the idea of broadband beam-forming in the frequency domain might seem more complex than beam-forming in the time domain, the effectiveness of FFT algorithms places lower demands on computer performance and operating memory capacity. In view of this, the modernized SQR-19PG sonar’s wideband beam-former is implemented in the frequency domain.

The design of the SQR-19PG sonar array is divided into sections based on frequency bands which are one octave wide [45]. One exception is VLF, the lowest frequency band, which comprises as many as 4 octaves. The length of the individual sections and the relevant frequency bands are shown in Tab. 2. The particular bands in beam-former algorithms follow a different process resolution whilst maintaining an identical resolution to bandwidth ratio. The VLF band is an exception, because for frequencies below 88Hz, due to the maximum array length and process resolution, its directional properties deteriorate.

Because of the design and principle of operation of sonar, a different beam width is obtained for each frequency line. Yet, the widths are identical in all frequency bands for lower, central and upper frequencies. Fig. 4 shows the central (non-deflected) array beam for lower, central and upper frequency of each band. In the other frequencies the widths of the central beam change almost linearly in the frequency function from 2.3° to 4.5°. In the VLF band, below 88 kHz, beam width increases significantly as frequency decreases. Fig. 5 shows central beam widths for very low frequencies.
Beam width also increases significantly for high deflection angles [45, 54]. This effect typically occurs in all beam-formers with linear array and cannot be eliminated. Maximum widths occur when beams deflect by +/- 90°, as shown in Fig. 6. For a maximum beam deflection, its width at –3 dB for the lower cut-off frequency is equal to 32°.

Depending on the beam-forming algorithm, the beams have a linear or non-linear distribution in the function of the deflection angle. When the FFT algorithm is used for spatial frequencies, adjacent beams intersect at a constant sensitivity level but the distribution of their directions is non-linear, as shown in Fig. 7. This is a feature of FFT beam-formers which becomes particularly relevant for large widths of the observation sector. If the beams are to have a linear distribution, the beam-former algorithm should use a non-linear phase shift distribution, which effectively means that the FFT algorithm cannot be used. The result will be a linear beam distribution as illustrated in Fig. 8. The computational effectiveness, however, is not as good as that of the FFT algorithm.

Tab. 2. Frequency bands of the SQR-19PG beam-former

<table>
<thead>
<tr>
<th>Band name</th>
<th>Section length [m]</th>
<th>Lower frequency limit [Hz]</th>
<th>Upper frequency limit [Hz]</th>
<th>Bandwidth [Hz]</th>
<th>Process resolution [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HF</td>
<td>24</td>
<td>701</td>
<td>1400</td>
<td>700</td>
<td>4</td>
</tr>
<tr>
<td>MF</td>
<td>48</td>
<td>351</td>
<td>700</td>
<td>350</td>
<td>2</td>
</tr>
<tr>
<td>LF</td>
<td>96</td>
<td>176</td>
<td>350</td>
<td>175</td>
<td>1</td>
</tr>
<tr>
<td>VLF</td>
<td>192</td>
<td>88 (10^3)</td>
<td>175</td>
<td>175</td>
<td>0.5</td>
</tr>
</tbody>
</table>

*) The real lower cut-off frequency.

Fig. 4. Central sonar beams for lower, central and upper frequencies

Fig. 5. Central beams for very low frequencies

Fig. 6. Beam patterns for beams deflected by 90°

Fig. 7. Sonar beams intersecting at a constant sensitivity level

Fig. 8. Equally spaced sonar beams
The beam distribution in the modernized SQR-19PG sonar is linear. 91 beams are generated in the beam-former covering a 180° observation sector, equally spaced every 2° (the entire real observation sector is 360° with ambiguous left right direction). As a result, the narrowest beams intersect at –3 dB, while deflected beams intersect at systematically higher levels. In addition, the resulting imaging scale is uniform and the scale is given in integers, which helps with interpretation and ranging.

The beam-former algorithms applied in SQR-19PG modernization are very complex – the computations are made separately for each frequency line and the linear beam distribution meant that the FFT beam-forming algorithm could not be used. Despite that, because the frequencies of the signals were relatively low, the beam-former was implemented on an industrial general-purpose COTS computer.

**Beam-forming for sonar with cylindrical array**

MG-322DSP and SQS-56PG type, hull-mounted sonars have multi-element cylindrical arrays. For the purposes of horizontal beam forming, the cylindrical array may be replaced with a simpler circular array whose individual elements represent the columns of a real array. If the width of each column is significantly lower than wave length, the columns may be replaced with “point” hydrophones, to keep it simple.

![Simplified cross-section of cylindrical array](image)

**Fig. 9. Simplified cross-section of cylindrical array**

The geometric centres of array elements are spaced at constant angular distances and the total number of columns is equal to M (30 for the MG-322DSP or 36 for the SQS-56PG sonar). A beam is generated in a cylinder sector which contains hydrophones marked from 1 to N in that figure. The next cylinder sector to generate the next beam is one array column from the previous sector. As a result, beams cover the complete angle of observation. The number of beams generated in such beam-formers is identical to the number of array columns. If the neighbouring beams are to intersect at –3 dB with regard to the acoustic axis level, then beam width is equal to angle α marked in Fig. 9. Beam-forming in a cylindrical array involves compensating for signal delays. They are the result of the different distances covered by waves coming to the array columns in a sector [5, 41, 54].

Unlike in the above described method, the modernized sonar with cylindrical array has a higher number of beams. In the case of the MG-322DSP sonar a cylinder sector containing 11 columns generates 3 beams. The central column’s centre of sectors determines the direction of the central beam. The axes of two neighbouring beams (left and right) are deflected from the central axis centre by –4° and +4°. The objective of phase compensation, in this case, is, for the acoustic wave incident on the sector’s central column, to generate identical phase shifts of all signals from the outputs of the array’s 11 columns. In the case of beams deflected by –4° and +4°, identical phase shifts have signals caused by acoustic wave incident on the array’s central column at –4° or +4°, respectively, with regard to perpendicular direction towards the central column. This is applied to the other cylinder sectors, each moved by one column, until 90 beams are generated, spaced every 4°. By using a similar method, the SQS-56PG sonar’s 36 columns generate 72 beams, spaced every 5°.

In a multi-beam spatial filter, phase compensation is done by multiplying cophasal and quadrature samples by numerical ratios. The operations produce real and imaginary parts of signals of sonar receiving beams. A complete phase compensation in the spatial filter occurs for the mid-channel frequency of sounding signal spectrum. This approximation has a minor effect on detection conditions, causing a slight increase in side lobe level of the patterns of the individual beams [52, 53].

The objective of the numerical ratios is for signal sample multiplication to perform amplitude weighing as well as phase shifts. This is to reduce the final side lobe level. A weighing “cosine on a pedestal” function was used with pedestal value equal to 0.4. Thanks to this, the resulting side lobe level was reduced to –18 dB. Fig. 10 shows the beam pattern of a single beam in the modernized MG-322DSP sonar. Fig. 11 shows the layout of all of the 90 receiving beams.

**Matched filtering in time domain**

In many of the modernized sonars the used signals have linear or hyperbolic frequency modulation with a high product of time duration and frequency bandwidth. If used in combination with matched filtering, the signals may improve the signal-to-noise ratio and detection range, which is particularly important for long range sonar. Digital processing methods offer several different ways of implementing matched filtering. The simplest and most computationally effective method is implemented in the frequency domain by using FFT algorithms. Echo signal samples from the entire range of distances (from the moment a sounding signal is sent until the echo signal is received from the end of the range) are converted into spectral form using FFT. The resulting complex spectral lines are multiplied by
complex values of conjugate spectral lines of the sounding signal pattern. Reverse FFT gives the result of echo signal matched filtering. There is a major downside to this method, however, because the computation does not start until echo signal samples from the entire range are collected. In the case of sonar with a range of some fifty kilometres it may take up to a minute before observation results are known. For this reason it is advisable to use a different method which will allow on-going detection as echo signals return. This can be done by using correlational detection in the time domain or the wavelet transform method [54, 58].

In the modernized MG-322DSP sonar, cophasal and quadrature signals from 90 outputs of a multi-beam spatial filter undergo correlational digital detection in a block of signal processors [57]. Correlational detection is made on complex signal samples from the outputs of the multi-beam spatial filter with cophasal samples treated as real and quadrature samples taken as imaginary. Processors compute in real time the functions of echo signal correlations. The number of complex echo signal samples depends on the sounding pulse duration and ranges from 213 samples for a 50 ms pulse do 6801 samples for a 1.6 s pulse. In order to determine one correlation signal sample, for each of the 90 beams we carry out from 213 complex multiplications and 212 summations of complex numbers (for a 50 ms pulse) to 6801 complex multiplications and 6800 summations of complex numbers (for a 1.6 s pulse). The operations take less than 230 μs. Thanks to the used solution, we can have a running display of signals after detection from 90 beams as echo signals are coming in.

Following correlational detection, the sounding signal is compressed in time by the product of bandwidth and signal duration. Fig.12 shows an enlarged shape of a sounding pulse after correlational detection for an LFM sounding signal and bandwidth of 800 Hz. The duration of the sounding pulse is 1s, i.e. 800 times longer.

**Method for improving multi-beam sonar bearing accuracy**

When a target is detected, operators of the modernized sonars may switch on the tracking mode, which further improves target positioning accuracy. To that end algorithms were developed at the Gdansk University of Technology whose origins go back to the mono-pulse method known from analogue technology [54, 59, 60].

Tracking can only be switched - on once the target is marked with a special marker. Placing a marker on a beam selects that particular beam and two neighbouring beams. The three beams are searched for signal maxima in the time interval around the marker. Next, three sequences are made of quadrature samples collected from array elements which generate the selected beams. The sequences are contained in double the duration of echo from the object being tracked. The samples are sent to the input of the tracking beam-former. It generates three beams: central, deflected to the left and deflected to the right. The beam-former input has three matched filters. Their signals are used to control beam rotation. The amplitudes of signals from two deflected beams are compared. In traditional systems if the signal from the left beam is bigger than the signal from the right beam, the beam-former deflects all beams to the left. Otherwise, it does so to the right. The target tracking beam-former deflects beams by changing the values of beam-former ratios. These are multiplied by complex signal samples to obtain equal signals in both beams. Beam deflection will continue until the signals of both beams are equal, i.e. when the absolute difference between them reaches minimum value. The angle at which signal difference reaches minimum value is the wave arrival angle. By tracking the signal from the central beam, we can improve beam control. Beams move at a step of 0.1°, which ensures the theoretical accuracy of angle measurement which is equal to the value of the step. This complex procedure is replaced with an algorithm
for determining direction based on the proportion between signals from neighbouring beams, a method developed at the Gdansk University of Technology [21, 22].

The practical accuracy of bearing depends on the signal-to-noise ratio which has an effect on the error in signal amplitude identification. Fig. 13 explains how an amplitude measurement error affects the angle measurement error. The figure also shows why the same amplitude measurement error generates a bigger error in direction when the beam pattern is used.

**MODERN VISUALIZATION METHODS**

The modernized sonars were equipped with modern and ergonomic displays with colour screen monitors, a wide range of imaging, display of settings, external data, messages, cursors, etc., which make up the so-called Man Machine Interface (MMI) [14, 42, 43, 44]. Sonar settings are simplified with new and modified settings supported with ergonomic pictograms and messages. Improvements were made to data transmission

![Fig. 14. Panoramic (PPI) imaging in the OKA-2M/Z sonar in active mode](image1)

![Fig. 15. Basic imaging in the mine counter-measure sonar MG-89DSP](image2)
between sonar elements and to computer methods for recording signals and images. The new visualization methods help operators with target detection, classification and tracking. Examples of visualizations in the modernized sonars are shown in Fig. 14 - 18.

CONCLUSIONS

Although it has been somewhat constrained by the existing ultrasonic transducers, the modernization has in fact led to the development of completely new sonar systems. The new
signal processing methods combined with new system solutions have effectively improved the parameters of the sonars and, as a consequence, the ships’ warfare capability.

What needs to be emphasized is that the approach is very cost efficient. The costs of the modernization represent some 20% of the cost to purchase new sonars with similar parameters.

Apart from conducting modernization projects, in recent years the team of the Department of Marine Electronic Systems at the Gdansk University of Technology has been working on developing new hydro-acoustic systems. The new designs include a unique silent sonar, which is difficult to be detected by enemy intercepting systems [33, 34, 35, 36, 38, 39, 55, 56].

With the new knowledge and experience gained by the Department of Marine Electronic Systems, the researchers are able to design and build other modern hydro-acoustic systems for naval applications, including multi-frequency and high resolution systems and sonars with synthetic aperture.

BIBLIOGRAPHY

Rationalization of servicing reefer containers in sea port area with taking into account risk influence

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ABSTRACT

This paper is aimed at presentation of a set of methods for rationalization of servicing containerized refrigerated cargoes in sea ports. During servicing reefer containers in port container terminals, take place various combinations of risk factors which can lead to loss of quality merits of cargoes contained in them. In the paper the risk factors capable of influencing quality of cargoes during their servicing in sea port, are identified, and the elaborated model for assessing risk level with the use of assumptions of fuzzy logic theory, is presented. Also, a simulation of servicing reefer containers in port, was performed. Moreover a prototype of an expert system which makes it possible to take correct decisions on servicing reefer containers in sea port, depending on impact level of risk factors, was proposed.

Keywords: sea port; reefer container; refrigerated cargo; risk factors

INTRODUCTION

Servicing reefer containers in sea port area belongs to a group of logistic tasks. Apart from loading, storing and intra-port transporting, in servicing cargoes of the kind such operations as: switching-on the container to an electricity source, veterinary and custom inspection, monitoring climatic conditions inside the reefer container during its stay in port etc. occur [5, 10, 11]. Specificity of fast spoiling cargoes forces to use another approach to servicing reefer containers in port than that for conventional containers. Refrigerated cargo is susceptible to change in climatic conditions during storing and transporting, hence it requires special care [3, 9, 13]. Extraordinary situations occurring in port area may lead to loss of quality merits (QM) of cargo, which in consequence may negatively influence repute of the port [4, 6, 10, 11]. Such problems take place in different ports worldwide, including the Baltic and Black Sea ports, where a degree of automation of loading operations is rather not very high: unattended container handling vehicles are not there in use, lack of a system for continuous remote monitoring operation parameters of reefer containers, etc. In order to prevent situations of the kind and take, in proper time, a correct decision against possible loss of quality of containerized reefer cargo (CRC), possible occurrence and effects of different combinations of risk factors should be carefully analysed. This paper is aimed at presenting a proposal of a method for rationalization of servicing reefer containers in sea port. Its scientific aspect is elaboration a model for assessment of risk level of loss of CRC quality merits, by using fuzzy logic theory.

LINKS OF CHAIN OF SERVICING CONTAINERIZED REFRIGERATED CARGOES IN SEA PORT AREA

To elaborate possible combinations of chains of servicing containerized refrigerated cargoes in sea port, should be collected in advance necessary data dealing with, a. o., a system of container servicing in port and elaborated a series of data bases to which the following belong:
- Characteristics of fast spoiling cargoes.
- Characteristics of reefer containers.
- Characteristics of reefer - container - carrying ships.
- Characteristics of infra- and supra- structure of sea ports.
- Particular links of chains of servicing containers in port area.
- Legal acts on transport and storage of food cargoes.
- Regulations and customs binding in ports etc.

In Tab. 1. an example of elements of the data base on “Particular links of chains of servicing containers in port area”, is presented. The links were completed on the basis of the data dealing with Polish and Ukrainian sea ports.

Main and auxiliary links of chains of servicing the containers in ports can be distinguished. The main links can be met almost in every port which renders services dealing with the servicing of containers. The auxiliary links (marked “*” in Tab.1) may not appear in CRC servicing chains in certain ports. The links serve for performing additional operations associated with servicing the container on request of a forwarder or needs of the port. In this paper have been analyzed the main links of container servicing chains, which play crucial role in their functioning.
**Tab. 1. Particular links of chains of servicing containers in port area**

<table>
<thead>
<tr>
<th>No.</th>
<th>Link</th>
<th>Illustration</th>
<th>Goal</th>
</tr>
</thead>
</table>
| A   | Unloading the container at ship side                                 | ![Unloading the container at ship side](image1.jpg)                         | - Relocation of the container from the ship on to quay or a land transport facility (unloading).  
    | Pre-B                                                                |                                                                              | - Relocation of the container from quay or a land transport facility on to ship (loading).       |
| B   | Temporary location of the container on to quay                       | ![Temporary location of the container on to quay](image2.jpg)               | - Short-time storing the container on quay at ship side, aimed at waiting for further relocation. |
| C   | Reloading the container over the port area                           | ![Reloading the container over the port area](image3.jpg)                   | - Relocation of the container in the relations of:  
    | Pre-P                                                                |                                                                              |   - Store place - car.  
    |                                                                        |                                                                              |   - Store place - lorry (or in opposite relations) etc.                                   |
| D   | Intra-port transport of the container                                | ![Intra-port transport of the container](image4.jpg)                        | - Relocation of the container between port store places or dedicated port areas, by means of port tractor. |
| E   | Relocation of the container over a store place area \(^1\) (reloading/ transport) | ![Relocation of the container over a store place area](image5.jpg)          | - Relocation of the container over a store place area to make it possible to reload containers placed in lower tiers.  
    | Pre-Pw-P                                                             |                                                                              | - Relocation of the container over a terminal area by means of port vehicles.                |
| F   | Storing/keeping the container on a store place                       | ![Storing/keeping the container on a store place](image6.jpg)               | - Location of the container on to a store place and connecting it to a power source.  
    | SkilI                                                                |                                                                              | - Control of external state of the container and recording CRC storage parameters.           |
| G   | Storing/keeping the container on a temporary store place             | ![Storing/keeping the container on a temporary store place](image7.jpg)     | - Temporary relocation of the container in order to perform different control operations or wait for further servicing. |
| H   | Veterinary control of CRC                                            | ![Veterinary control of CRC](image8.jpg)                                    | - Inspection of content of the container by a veterinary doctor. It comprises:  
    | KV                                                                   |                                                                              |   - Opening the container door.  
    |                                                                        |                                                                              |   - Taking cargo samples.  
    |                                                                        |                                                                              |   - Closing the container and sealing it with lead.  
    |                                                                        |                                                                              |   - Issuing a quality certificate for CRC.                                                   |
| I   | Custom inspection and frontier control of CRC                        | ![Custom inspection and frontier control of CRC](image9.jpg)               | - Examination of container content by custom and frontier service:  
    | KCG                                                                  |                                                                              |   - Opening the container doors.  
    |                                                                        |                                                                              |   - Examination of container content.  
    |                                                                        |                                                                              |   - Closing the container and sealing it with lead.  
    |                                                                        |                                                                              |   - Issuing documents.                                                                      |
During designing transport systems of any kind is performed an analysis of development of every situation in which the system in question can be used, including extraordinary ones. Any hazard is very tightly connected with the notion of risk which is usually defined as the product of occurrence frequency (or probability) of a given hazardous event within certain time interval and consequences associated with the event [1, 12, 14]. The risk of loss of CRC quality merits can be calculated by using the formula (1).

\[
P(V) \cdot K(V) = R \leq R_{	ext{akc}}
\]

where:
- \( P(V) \) – probability of loss of QM of CRC \([0 \div 1]\),
- \( K(V) \) – amount of loss in the case of loss of QM of CRC (it depends on a kind of containerized cargo) \([\text{zł}]\) (Polish currency),
- \( V \) – conditions of realization of CRC servicing,
- \( R \) – calculated risk \([\text{zł}]\),
- \( R_{	ext{akc}} \) – acceptable risk \([\text{zł}]\).

Number of combinations of risk factors which result in extraordinary situations in port area is very large [4, 6, 7, 8], therefore all the risk factors are proposed to be reduced to the groups presented in Tab. 2.

As results from Tab. 2, the human factors constitute a group of factors most often appearing in port area. Analyzing the methods for assessing risk level one can state that human behaviour is hard to be assessed by using traditional methods (e.g. probability theory) as man thinks in a fuzzy way. Hence in this paper the assumptions of fuzzy logic theory were selected for assessing all the factors contributing in loss of QM of CRC in port area. The theory makes it possible to assess objects and processes in which input data are not precisely determined. Because in the case of loss of CRC quality merits the amount of losses \( K(V) \) (i.e. the cost of servicing a given CRC) is known and determined, in this paper the risk assessment (PR) was mainly based on estimation of certainty level of losing CRC quality merits, calculated with the use of the fuzzy measure of possibility \( \text{Poss} (V) \) \([0 \div 1]\).
For elaboration of the model of risk level assessment the following assumptions and limitations were taken:

**A) Assumptions:**
1. The risk factors defined and split into three groups, characterize real conditions of servicing containers in sea ports and impact CRCs independently.
2. Occurrence of a combination of different risk factors is possible.
3. Risk factors have linear dynamics of impact on to CRCs.

**B) Limitations:**
1. Duration time of servicing CRC in port area does not exceed expiry time of the CRC.
2. For analysing a separate link it is assumed that risk factor duration time does not exceed duration time of the link in question.

To perform a quantitative assessment of risk level of CRC QM loss a calculation model was elaborated. The model is aimed at the assessing of risk level both during performing particular links of container servicing chains and performing the whole chains. Fig. 1 shows the model’s structure. In Fig. 2 is presented the author’s conceptual model which covers data mutual relations used for assessing risk level of CRC QM loss. It was elaborated on the basis of the data acquired from sea ports and saved in data bases. The conceptual model’s structure consists of the peaks and directed connections between them, which define a set of decision making criteria.

**MODEL OF RISK LEVEL ASSESSMENT**

For elaboration of the model of risk level assessment the following assumptions and limitations were taken:

**A) Assumptions:**
1. The risk factors defined and split into three groups, characterize real conditions of servicing containers in sea ports and impact CRCs independently.
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3. Risk factors have linear dynamics of impact on to CRCs.

**B) Limitations:**
1. Duration time of servicing CRC in port area does not exceed expiry time of the CRC.

<table>
<thead>
<tr>
<th>Group of factors</th>
<th>Risk factors</th>
<th>Frequency of occurrence [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human factors (subjective)</td>
<td>- Individual psychological and physiological factors (e.g. state of illness or tire etc)</td>
<td>60 - 70</td>
</tr>
<tr>
<td></td>
<td>- Individual professional factors (e.g. insufficient qualifications etc)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Factors of improper organization of servicing CRC (e.g. lack of instruction for servicing given cargoes, insufficient number of employees etc)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Collective factors resulting from multi-nationality and lack of knowledge of communication language (e.g. improper information exchange, unprecisely attributed tasks etc)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Social factors: strikes, epidemies, thefts, terrorism etc.</td>
<td></td>
</tr>
<tr>
<td>Technical and technological factors (objective-subjective)</td>
<td>- Unsatisfactory state of port infrastructure (e.g. improper state of intra-port roads, store places etc)</td>
<td>25 - 35</td>
</tr>
<tr>
<td></td>
<td>- Unsatisfactory state of port supra-structure (e.g., unreliability of cargo handling facilities, intra- port transport means etc)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Unsatisfactory state of reefer container (e.g. failures of elements of refrigerating units and defects of reefer container box etc)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Improper technique of servicing the container (e.g. lack of an appropriate number of supra-structure facilities)</td>
<td></td>
</tr>
<tr>
<td>Natural climatic factors (objective)</td>
<td>- Unfavourable climatic and aerologic conditions (e.g. strong winds, snowfall, rains, hurricanes, tornados etc)</td>
<td>5 - 15</td>
</tr>
<tr>
<td></td>
<td>- Unfavourable hydrological conditions (e.g. tsunami, waving, flooding etc)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Unfavourable seismic conditions (e.g. earthquakes etc)</td>
<td></td>
</tr>
</tbody>
</table>

*) expert assessment, the data are different for each of the ports

**Source:** the author’s original elaboration based on an inquiry action conducted among experts.

![Fig. 1. Structure of the calculation model of risk level assessment of CRC QM loss during designing rational chains of cargo servicing in sea ports. Source: the author’s original elaboration](image-url)
of rules for transition of input data to calculation results. The connections are considered to be relations which describe influence of one category of data on other ones.

The peaks of the 1st layer of the analysed model are called the model’s characteristics. They describe state parameters of reefer container serviced in port area. The characteristics are determined by sets of gradations (Tab.3), which form sets of input data and are formalized by scales of possible changes of their particular values.

As known from CRC servicing practice, impact of risk factors onto the cargoes is time-varying. The fact has been taken into account in the proposed model. In the model, risk factors change values of characteristics of research objects, that consequently has influence on change of integral values in particular layers of the model. By accounting for the influence a dynamics which reveals functioning the CRC servicing chains, is formed.

In order to obtain the correct functioning of the calculation model, consecutive data bases have been elaborated to be used to:
- assigning values (fuzzy measures) in particular peaks of the conceptual model,
- determining values for gradation of characteristics of research objects,
- determining intensity and duration time of impact of risk factors etc.

![Tab. 3. Gradations of particular characteristics of the conceptual model](image)

![Fig. 2. Information data interrelations in the conceptual model, source: the author’s original elaboration](image)
To build the data bases was used information achieved from expert inquiries, rules, standards and legal regulations as well as that gained by this author during her training periods spent in sea ports of Poland and Ukraine.

The calculation algorithm of risk level during realization of a single chain link of container servicing is presented in Fig. 3. In the model, to perform calculations, the assumptions of fuzzy logic theory were used [2, 8]. The calculations were carried out by using an adjusted software based on the complex ExproMaster 6.0, whose results delivered risk levels of loss of CRC quality merits (Fig. 4).

In the process of the model verification a satisfactory convergence of statistical data with those obtained from the calculations, was reached.

**MAKING DECISIONS DURING DESIGNING THE CHAINS OF SERVICING THE CONTAINERIZED REFRIGERATING CARGOES IN COMPLIANCE WITH THE RISK LEVEL MINIMIZING CRITERION**

After performing the assessment of risk level of CRC QM loss in particular links and during realization the whole servicing chain, decision dealing with final form of the servicing chain of such cargoes in port area, should be made. Example ranges which classify risk levels of CRC QM loss, are presented in Tab. 4.

If the risk level of QM loss of a serviced refrigerated cargo is contained in the acceptable range (PR ≤ 0.2), decision-making person is able to accept servicing the cargo in accordance with decisions taken during preliminary design process of a given...
### Tab. 4. Example ranges which classify risk levels of CRC QM loss, based on the experts’ inquiry action

<table>
<thead>
<tr>
<th>Example range of risk level</th>
<th>Risk level of CRC QM loss</th>
<th>Acceptable risk level of CRC QM loss</th>
<th>State of refrigerated cargo</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; PR_{akc} ≤ 0.03</td>
<td>minute</td>
<td>ideal</td>
<td>- cargo maintains its initial merits and can be sold</td>
</tr>
<tr>
<td>0.03 &lt; PR_{akc} ≤ 0.1</td>
<td>insignificant</td>
<td>acceptable</td>
<td>- cargo maintains its initial merits, but insignificant changes of its transport conditions occur</td>
</tr>
<tr>
<td>0.1 &lt; PR_{akc} ≤ 0.2</td>
<td>low</td>
<td>acceptable</td>
<td>- cargo maintains its initial merits but is transported in improper conditions</td>
</tr>
<tr>
<td>0.2 &lt; PR_{akc} ≤ 0.4</td>
<td>limiting</td>
<td>acceptable provided the decision-making person takes responsibility for it</td>
<td>- short-lasting changes in recommended cargo storing conditions occur</td>
</tr>
<tr>
<td>0.4 &lt; PR ≤ 0.6</td>
<td>critical</td>
<td>conditionally acceptable</td>
<td>- cargo is exposed to loss of its quality</td>
</tr>
<tr>
<td>0.6 &lt; PR ≤ 1.0</td>
<td>disastrous</td>
<td>inacceptable</td>
<td>- cargo is especially exposed to loss of its quality or spoiling</td>
</tr>
</tbody>
</table>

PR – risk level of CRC QM loss; PR_{akc} – acceptable risk level of CRC QM loss

**Source**: the author’s original elaboration

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**Fig. 4.** Dialogue windows showing particular calculation phases: a) dialogue window for inserting fuzzy measures into gradations of characteristics of examined objects; b) dialogue window for inserting power of impact of risk factor onto object of examination; c) dialogue window showing temporary results of calculations in peaks of 2nd layer of the conceptual model; d) dialogue window showing results of calculation of integral risk level of loss of refrigerated cargo quality merits. **Source**: the author’s original elaboration
chain of CRC servicing, and not to take any action connected with its rationalization. If the PR is located in the range of the limiting risk level ($0.2 < PR \leq 0.4$), decision-making person may undertake realization of a given link of CRC servicing chain on his personal responsibility.

If the aforementioned risk level is located in the conditionally acceptable range ($0.4 < PR \leq 0.6$), decision-making person may accept CRC servicing in accordance with the prior taken decisions (i.e. on his personal responsibility) or introduce changes into the servicing process of the cargo in question.

If decision – making person chooses the direction of introducing changes into chain of CRC servicing then it will be possible to introduce them into:
- structure of to-be-used elements of infra- and supra-structure of a port,
- structure of organization system of port servicing,
- structure of cargo servicing technology (changes in number and sequence of links put in the CRC servicing chain).

If the risk level during servicing the CRC exceeds the conditionally acceptable level ($PR > 0.6$), decision-making person should resign himself to service a given cargo in a sea port or undertake definite actions to lower the risk level or - by means of negotiations - to change initial assumptions as to servicing the container in sea port. Decisions made at a given risk level should be aided by additional resources (infrastructural, financial etc).

### RESULTS OF SIMULATION INVESTIGATIONS

On the basis of the presented calculation algorithm (Fig. 3) were performed calculations of risk levels of loss of CRC QM during realization of servicing chains of a given CRC in given conditions in a selected port „X”. The calculations were carried out for three scenarios of impact of risk factors (optimistic, moderate and pessimistic one). Tab. 5 shows results of the calculations for each of four selected representative servicing

---

<table>
<thead>
<tr>
<th>No.</th>
<th>Risk level during realization of particular servicing link of a selected CRC</th>
<th>Risk level during realization of the chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.22 0.11 0.20 0.19 0.10</td>
<td>0.22 (Limiting)</td>
</tr>
<tr>
<td>2</td>
<td>0.19 0.10 0.12 0.15 0.17 0.11 0.19</td>
<td>0.19 (Low)</td>
</tr>
<tr>
<td>3</td>
<td>0.15 0.08 0.16 0.10 0.21 0.14 0.24</td>
<td>0.24 (Limiting)</td>
</tr>
<tr>
<td>4</td>
<td>0.15 0.08 0.16 0.13 0.12 0.10 0.19</td>
<td>0.22 (Limiting)</td>
</tr>
</tbody>
</table>

Source: the author’s original elaboration

---

Tab. 5. Results of assessment of risk level during realization of particular links and the whole servicing chain of a selected CRC, acc. the moderate scenario

![Fig. 5. Results of assessment of risk levels during realization of the investigated chains of CRC servicing in a port „X”, acc. three scenarios of impact of risk factors. Source: the author’s original elaboration](image-url)
chains of a given CRC \[10, 11\] in compliance with the moderate scenario. In the table colours of the links correspond to colours and names of the links in Tab. 1. Combination of the obtained results of risk level calculations for the three scenarios, is given in Fig. 5.

On the basis of the obtained data it is possible to determine variants of change of risk level of CRC QM loss depending on intensity and duration time of impact of risk factors.

In Fig. 6 is presented the dependence of risk level on duration time of realization of the link „Storing the container on store place” for five variants of container servicing.

As results from the diagrams in Fig. 6, risk level of CRC QM loss changes in the range of \(5 \div 40\%\), depending on a way of realization of CRC servicing link.

With the use of the elaborated model was determined a.o. the dependence of risk level of CRC QM loss on reefer container’s age of operation (Fig. 7).

As a result of calculations was also obtained dynamics of change in risk level of CRC QM loss, resulting from the selected groups of risk factors (Fig. 8).

As results from Fig. 8, in the case of realization of the chain ”1” (Tab. 5) for the moderate scenario, impact of human factors on risk level exceeds impacts of the remaining groups of risk factors by \(10\div 15\%\). This demonstrates that behaviour of human being and its errors greatly contribute to possible occurrence of extraordinary situations. The decisions made for CRC servicing should hence take into account the intensity and duration time of impact of subjective factors on realization of both particular links and the whole cargo servicing chain in port.

On the basis of the performed analyses of the above mentioned cargo servicing chains the following conclusions can be offered:

1. Change of sequence of realization of links of CRC servicing chains can lead to a change of their weakest links, i.e. occurrence of the highest risk level in another link.

2. There are two ways of lowering the risk level of CRC QM loss: - to introduce changes in realization of the weakest link (-s) of CRC servicing chain or - to choose another chain of servicing the cargo. In the first case the risk level may be lowered by \(10\div 18\%\), in the other - by \(8\div 21\%\). As

\[\text{Fig. 6. Dependence of risk level on duration time of realization of the link „Storing the container in store place”}. \quad PR \text{ – risk level of CRC QM loss, } \tau \text{ – duration time of realization of the link „Storing the container in store place”, } 1 \text{ – the container was stored in store place in 1st tier, } 2 \text{ – the container was stored in store place in 2nd tier during hot weather, } 3 \text{ – the container was relocated several times over store place, } 4 \text{ – custom control was performed in store place (the container’ s door was opened), } 5 \text{ – the container was left for a long time without electric supply in store place. Source: the author’s original elaboration} \]

\[\text{Fig. 7. Dependence of risk level of CRC QM loss on reefer container’s age of operation}. \quad PR \text{ – risk level of CRC QM loss, } \tau \text{ – reefer container’s age, } a \text{ – function of risk level versus reefer container’s age, } b \text{ and } c \text{ – upper and lower variation interval of uncertainty of the function } a, \text{ respectively. Source: the author’s original elaboration} \]

\[\text{Fig. 8. Dynamics of change in risk level of CRC QM loss for different groups of risk factors during realization of the chain „1” acc. the moderate scenario. } \quad PR \text{ – risk level of CRC QM loss, } \tau \text{ – duration time of realization of the link „Storing the container in store place”, } 1 \text{ – impact of human factors, } 2 \text{ – impact of technical and technological factors, } 3 \text{ – impact of natural climatic factors. Source: the author’s original elaboration} \]

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results from this, it is important to introduce changes both into realization of the weakest link and sequence of links of the CRC servicing chain. Change in sequence of realization of links results as a rule in a lower financial expenditure.

3. Because values of the calculated risk levels during realization of particular links of the analysed chains are rather close to each other, it is not always possible to identify the weakest link on realization conditions of which the integral risk level of the whole chain depends. Therefore it can be stated that in majority of cases decisive impact on integral risk level of servicing chain is associated with number and sequence of its realization links.

4. As a result of the performed investigations it can be stated that, disregarding situations connected with accidents and damages of reefer containers, the fact of disconnection of the container from electric supply is of the greatest detrimental effect on CRC QM loss [6], regardless of kind of risk factors causing a given situation. As number and duration time of the disconnection events depend on number of links of CRC servicing chain, sequence of their realization as well as port infra- and supra-structure engaged in it, cause-effect relation between the CRC QM loss and sequence and way of carrying out operation of servicing the cargo in port area, is this way revealed.

5. Simultaneous realization of certain operations, e.g. custom and veterinary control, decreases duration time of realization of link (-s), that consequently results in lowering the risk level. It seems that the problem of coordination of operations of all the services taking part in CRC servicing in the organizational, technical and technological aspects is presently more and more widely observed as an active dialogue between scientists, port personnel, administration officials and cargo owners has been initiated [5].

**SUMMARY**

The presented set of methods constitutes a prototype of an expert system aimed at generating alternative variants of servicing reefer containers in sea port area, and making their subsequent comparisons.

Tendency of sea ports towards rationalization of cargo servicing in their areas is associated not only with improvement of their infra- and supra-structure but also creation of suitable conditions for rational decision-making by entitled persons. Activity of ports should be directed towards elaboration and implementation of the systems for rational decision-making by entitled persons. Investment into the systems (a.o. expert systems) for aiding decisions to be taken should make it possible to improve quality of CRC servicing as well as reduce risk of CRC’s QM loss in port area.

The set of methods for rationalization of CRC servicing in sea ports, proposed by this author, makes it possible to form different CRC servicing chains and compare them with taking into account risk factors affecting quality merits of the cargoes. As results from the performed analysis, safety of refrigerated cargo during its servicing in sea port area depends both on a state of reefer container reliability and quality of its servicing by port personnel. Impact of human factors characterised by their variety and high frequency of occurrence, on quality of cargo during its servicing in port area, considerably exceeds impact of other risk factors. The application of fuzzy logic to analysing risk level of loss of quality merits of refrigerated cargoes is justified and makes it possible to assess impact of subjective factors onto CRC servicing.

The elaborated set of methods for rationalization of CRC servicing, based on computer software, makes it possible to improve operational effectiveness of sea port functioning in contemporary market conditions.

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Application of multi-criteria optimisation in marina planning on the Montenegrin coast

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ABSTRACT

Development of nautical tourism in Montenegro, seen as a potentially profitable industry segment has started twenty years ago. The first steps taken in this direction were defining the legal framework, in particular: Law on Coastal Zone, Tourism Master Plan of Montenegro, Law on Yachts, followed by establishment of supporting organizations - Maritime Safety Department of Montenegro, Port Authorities and Border Police Department. Having invested efforts in adoption of planning documents and enactment of legislation relating to nautical tourism in recent period, Montenegro is seeking to impose itself as an important nautical destination. The first serious step towards achieving this goal was construction of a world-class marina Porto Montenegro in Tivat in 2010. The Special Purpose Spatial Plan for Coastal Zone provides for development of network of marinas to be constructed on the Montenegrin Coast. In the light of this, contribution of this paper is reflected in application of multi-criteria optimization in defining an optimum order of marina construction. The paper defines selection of criterion functions from the aspects of ecology, economy, security, specific aspect and aspect of suitability assessment for the location. The paper also presents the process of defining different criterion weights. The multi-criteria analysis determines a compromise solution for marina planning at Montenegrin Coast, which reconciles conflicting interests. The compromise solution is only a proposal, while a final decision remains with a decision maker.

Key words: marina location; multi-criteria optimization; criterion functions; ecology

INTRODUCTION

The Montenegrin Coast represents a valuable complex of natural and man-made resources, being a backbone of the main branches of industry of the State, tourism industry in particular. With promulgation of the Law on Coastal Zone in 1992, Montenegro recognized and acknowledged the high importance of extraordinary values of the coastal area and defined frameworks for special preservation regime, utilization and improvement of this important resource. Adopting and implementing key development strategies as defined in Tourism Master Plan of Montenegro (2001), tourism industry is expected to provide directly receivable turnover at the amount of one billion Euros by 2020, provided that the tourism offer is aimed at increase of high-quality capacities.

Tourism branch can derive valuable benefits from development of its nautical segment through making the most of huge possibilities of the existing and new marinas [4, 5, 6]. Equipping marinas to the full extent is a condition precedent for development of nautical tourism. Motivation for writing this paper is to define an optimum order of marina construction on the Montenegrin Coast, otherwise the coast risks to become a sailing destination avoided by sailors, while huge profit will be lost to neighbouring countries. The Special Purpose Spatial Plan for Coastal Zone (2005) specifies locations intended for marina construction without prior necessary analysis of suitable micro-locations and optimum order of implementation. By applying the multi-criteria optimization a compromise solution for marina micro-locations out of the set of alternative locations can be defined, along with the ranking list of alternatives based on the predefined criteria. A finite set of alternatives is then presented to the decision-makers and forms ground for reaching a final decision. Each alternative should be evaluated in terms of each and every criterion. The compromise ranking method in multi-criteria decision making process (referred to as VIKOR) is formed on such methodological grounds which assume that a decision maker is presented with the alternatives representing a compromise between wishes and possibilities, or that reconciling conflicting interests of all relevant shareholders in the decision-making process.

EXISTING AND PLANNED CONDITION OF MARINAS ON THE MONTENEGRIN COAST

Apart from the world-class marina Porto Montenegro in Tivat, present conditions for storage and handling ships in the ports along the Montenegrin coastline are pretty modest. Marinas and ports to be used as mooring spots for sailing vessels are constructed in the coastal towns of Tivat, Bar,
Budva, Prcanj, Kotor, Herceg Novi, and Meljine, respectively. Project of Porto Montenegro in Tivat was initiated once the major investors lead by Peter Mank, president of the leading gold-mining corporation, had indentified a growing demand for yacht berths in the Mediterranean. Porto Montenegro was constructed in 2010, offering to its customers a full-scale service, from supply of spare parts to technical problem solving. The marina Porto Montenegro in Tivat is the most advantageous luxury yacht marina in the Mediterranean Sea, ranked among the world-class marinas. The marina in Bar has 665 berths on sea and 120 berths on land. The marina in Budva provides 400 for boats and 10 berths for larger vessels. Infrastructure connections are provided in part of the quay. The marina in Prcanj is the first one constructed on the Montenegro coast and financed with private capital. It is a small marina which represents certain pilot project of this type of tourist offer. Port of Kotor avails of an operative quay with the nautical infrastructure provided in the south part of the quay, the capacity of which is 10 to 15 vessels of different categories. Port of Herceg Novi is dominantly used for excursion boats operating to local destinations and fishing boats. Meljine has a breakwater constructed on its west side. The existing berths are used for local boats, and it can provide berths for 20 nautical vessels.

A key challenge for development of nautical tourism in Montenegro is the existence of not more than one high-level, fully equipped marina located in Tivat. The existing marinas in Bar, Budva and Prcanj need to be upgraded to a far higher level in terms of equipment and facilities required for provision of nautical services, while the remaining marinas are used as mooring spots for local boats only. The Special Purpose Spatial Plan for Coastal Zone (2005) foresees establishment of a 2000-berth network in Montenegro to be consisted of two large service marinas with the capacity of 400 to 500 berths, four standard marinas with the capacity of 100 to 300 berths, and four small marinas. The existing mattresses and old docks in the Bay of Boka can be used for the purpose of nautical tourism upon undergoing careful revitalization. Two large service marinas are planned to provide all necessary nautical facilities and other services to its customers, therefore a provision of area on land should be safeguarded for future expansion. Such world-class marina as the Porto Montenegro was built in Tivat in 2010 on the location of former Ship Overhaul Institute, while the other one should be constructed in Bar at the location of the existing marina. The standard four marinas should meet demand of nautical sailors at the key points along the Montenegrin Coast. In this context, marinas at the following locations should be constructed: Rt Kobilad, Liman in Ulcinj, byland Lustica in Bigovo, and in Kumbor. Construction of four small marinas is planned at locations of Ada Bojana, Buljarica, Budva and Kotor (along the Old Town). The commercial berths are planned in Njivice, Meljine, Zelenika, Prcanj and Bonici near Tivat, Sveti Nikola Island near Budva, and on the part of the coastline spreading from Rt Djerane to Porto Milena in Ulcinj.

**CRITERION FUNCTION IN MULTI-CRITERIA OPTIMIZATION**

Multi-criteria optimization is applied to determine the ideal alternative for marina locations out of the set of possible alternatives or the set of suitable alternatives. A criterion is expressed through a criterion function which is expected to reach a global extreme for the best alternative with regards to the limitations represented by capabilities of reaching a goal [1, 2]. In defining the criterion functions for marina micro-location all relevant elements of the system should be considered. The basic criteria applied for the purpose of comparison of alternatives comprise the following groups:

- environmental impact criteria,
- economic criterion,
- maritime security and safety criteria,
- specific criteria,
- assessment of location suitability criterion.

Special consideration is given to the environmental impact criteria which can be classified into two groups. The first group defines impact of marinas on the marine ecosystem, preservation of environment, with an aim to minimize effects to the flora and fauna of the aquatic ecosystem and to safeguard environment for future generations. This criterion function is expressed by the function of environmental risks $f_1$; while the second criterion function $f_2$ relates to preservation of other resources, impact on surrounding beaches and ambient values of the area. Both construction and operational phases of marinas should be aimed at prevention of environmental pollution by ships and protection of marine environment and intertidal zone.

General condition of the sea-shore quality is poorer in the closed sea (example: Bay of Boka) than in the open sea. The latest researches show that the Bay of Boka is exposed to intensive impact of human activities, the wastewaters have been observed to often cause excessive aquatic plant growth (algal bloom), while concrete biocenosis is being devastated. The evaluation of the environmental impact criteria is performed by environmental experts.

A 5-point scale is used for evaluation, where 1 stands for high environmental risk, 2 for moderate environmental risk, 3 for low environmental risk, 4 for no environmental risk, and 5 for positive impact of marina on the environment, e.g. improvement of the location planned for marina construction. Lower values are assigned to the marinas located in the Bay of Boka, somewhat higher to the marinas in the bays of Kotor and Herceg Novi, while the highest values are allocated to the marinas in the southern part of the coast.

The second group of criteria – the preservation of other resources, $f_2$, affects the surrounding beaches and ambient values of the area. According to the available data, the Montenegrin Coasts avails of more than 100 beaches in total length of 70.35 km (area of approximately 271.5ha) visited for swimming and sun tanning. The following beaches are located in the vicinity of the planned locations for marina construction: Njivice (1,800 m), Meljine (325 m), Zelenika (430 m), Kumbor (235 m), Kotor (320 m), Prcanj (2,820 m), Tivat (375 m), Bigovo (40 m), Budva (Avala – 340 m, Old Town – 230 m, and Slovenska plaza – 1.620 m), Sveti Nikola (575 m), Buljarica (2,350 m), Bar (Topolica – 750 m, Susanj – 870 m), Liman (105 m), Porto Milena (100 m) and Ada Bojana (2,750 m). The evaluation of the environmental impact criteria is determined by length in kilometres of those beaches which are near the planned marinas, with the aim to minimize this criterion function. The economic criterion $f_3$ is formulated as a criterion function of the marina construction costs in the currency of Euro. Within the scope of the economic evaluation there are procedures for determining costs of each alternative inclusive of the expropriation. Maritime safety and security criteria are determined by the criterion functions of the nautical conditions $f_4$; and maritime conditions $f_5$. The nautical conditions $f_4$; come as a result of consideration of the navigational safety along the coastline. To navigate along the part of the sea, the nautical sailors are informed in advance if there is sufficient number of marinas on the planned route for a case of unplanned and emergency stops. It is very important that the nautical sailors feel safe and that they can expect all necessary assistance and support and information in case of a defect or
damage, unfavourable weather conditions etc. Authorities in charge in case of a ship incident on the Montenegrin Coast are Maritime Safety Department of Montenegro, Port Authorities in Bar and Kotor, and Border Police Department. The evaluation of the criterion function of nautical conditions is performed based on the required response time (in minutes) in case of an accident at sea. The aim is to minimize this criterion function. The adopted response time ranges from 20 min for Marina Bar to 50 min for marinas located on the north and south ends of the coast (Rt Kobila and Ada Bojana, respectively). The unfavourable maritime conditions f5; have adverse effect on the maritime safety. The sea wave direction at the Montenegrin Coast is defined based on the recorded frequency at certain stations, with special consideration given to the calm sea periods. Based on the available data, periods of calm sea are recorded at the station in Kotor in duration of 57.7% of the year, at the station in Herceg Novi for 59.1% of the year, and at the station in Budva for 59.1% of the year, while the occurrence of calm sea periods has not been recorded at the stations in Bar and Ulcinj.

The evaluation of the maritime condition criterion is performed based on the calm sea period which will not affect the navigation period expressed in percent. The aim is to maximize this criterion function.

Specific criteria refer to specific features of the micro-locations intended for marina construction. The criteria are classified in two groups, the first deals with position and accessibility f6; and the second with project feasibility, f7. The criterion function of position and accessibility f6; is influenced by the level of current conditions and completeness level of the available transport infrastructure, and the distance from airport. Upon sea navigation, the sailors usually use road as well as air transport for their arrival and/or departure. When visiting Montenegro, sailors usually arrive at one of the two Montenegrin airports, i.e. Podgorica and Tivat Airports, and Dubrovnik Airport in Croatia. The road infrastructure is in poor conditions and there is no high-quality road connection with the neighbouring countries. The evaluation of the criterion function of position and accessibility is determined by the length of the road running from the airport to the planned marina location. The aim is to minimize this criterion function. The criterion function of project feasibility f7; relates to consideration of topographic conditions and ownership issues at micro-locations intended for marina construction.

The evaluation of the criterion function of project feasibility uses the 5-point scale, where 1 point is assigned for a low-level of feasibility due to topographic conditions and huge ownership issues, 2 points are assigned for a low-level of feasibility together with the ownership issue, 4 points are assigned for a high-level of feasibility with no ownership issues present, and 5 points are assigned for a high-feasibility project due to favourable topographic conditions (plain terrain, no rocks) and no ownership issues present.

The criterion of location suitability assessment expresses the attractiveness of the location for marina construction from the aspect of the expected demand for nautical berths f8. The marinas on the south and north ends of the Montenegrin Coast are deemed highly attractive for the said purpose, the former due to the vicinity of the Strait of Otranto which would ensure the entering of higher number of nautical vessels, while the latter due to the vicinity to the Croatian coast, known for high presence of nautical vessels. The marinas in the Bay of Boka are also deemed attractive locations due to natural and cultural values. The evaluation uses the 5-point rating scale where 1 stands for poor rating, 2 for fair rating, 3 for good rating, 4 for very good rating and 5 for excellent rating of the location from the aspect of attractiveness.

EVALUATION AND RANKING ALTERNATIVES

The VIKOR method is introduced in the process of evaluation and multi-criteria ranking [3] of the alternative locations for marina construction on the Montenegrin Coast. This method requires defined values of criterion functions for each alternative.

For the purpose of VIKOR algorithm:

Set of j alternatives is denoted as (a1, ..., aj), set of n group of criterion functions is denoted as (f1, ..., fn); fij is the value of the i-th criterion function for j-th alternative, wi is the weight of the i-th criterion function; v is the weight of the strategy of majority of the criteria, and Qj is the measure of multi-criteria ranking.

Multi-criteria ranking by VIKOR method is performed based on the Qj measure which can be of the following relation:

\[
Q_j = v (S_j - S^*)/ (S^* - S^*) + (1 - v) (R_j - R^*)/(R^* - R^*)
\]

where:

\[
S_j = \sum wi (f_i^* - f_j)/ (f_i^* - f_i^*)
\]

\[
R_j = \max wi (f_i^* - f_j)/ (f_i^* - f_i^*)
\]

The best values of the limit metrics S and R:

\[
S^* = \min_j S_j; R^* = \min_j R_j
\]

The worst values of the limit metrics are:

\[
S^- = \max_j S_j; R^- = \max_j R_j
\]

Qj measure can also be formulated as here below:

\[
Q_j = v Q S_j + (1-v) QR_j
\]

where:

\[
Q S_j = (S_j - S^*)/ (S^* - S^*)
\]

\[
Q R_j = (R_j - R^*)/ (R^* - R^*)
\]

Ranking of the alternatives is based on sorting by the values of the measures QS, QS and Q in decreasing order. The best alternative is the one with the minimum value of measure and this alternative will take first position in the ranking list. The alternative aj is better than the alternative ak if the condition Qj < Qk is satisfied. The obtained result is three ranking lists. The measure Qj is a linear function of the weight v of the decision making strategy ‘the majority of criteria’, therefore the rank in the Q list is a ‘linear combination’ of ranks in the lists QR and QS. By applying the Q-metrics a rank list of all considered alternatives, e.g. a compromise list, is obtained. If a decision-maker has not predefined values of the weights wi of the criterion functions, and the weights v of the strategy ‘the majority of criteria’, the initial solution can be considered without giving preference to any individual criterion, with introducing values of the weights wi = 1 and/or v = 0.5.

VIKOR method suggests that the best alternative in terms of multi-criteria evaluation (for defined weights wi) is the alternative best ranked in the compromise ranking list for v = 0.5, i.e. that having the acceptable advantage
and the acceptable stability in decision making. If the first ranked alternative in the compromise ranking list fails to satisfy both of the criteria, this alternative is deemed not to be better than the second ranked alternative. The acceptable advantage is defined by difference between the Qj measures between the first compared to the subsequent alternative for \( v = 0.5 \). In terms of the acceptable stability in decision making the compromise solution should also be ranked best in the QS or QR ranking list. In the course of the ranking process a decision-maker can vary the criterion weights \( w_i \), depending on the evaluation preferences given to certain criterion functions. Specifying weight criteria is a special issue in the multi-criteria optimization and represent input values for the VIKOR method. The compromise solution is reached through the following steps of the VIKOR algorithm: determination of ideal solution, transformation of varied criterion functions, determination of the criterion weight \( w_i \), determination of the criterion weight \( v \), determination of the measures \( S_j, R_j, Q_j \), \( j \ldots J \), ranking in terms of the values \( S, R, Q \). The compromise solution is a solution closest to the ideal solution based on the adopted distance to the ideal solution.

Tab. 1 shows values of the criterion functions for the multi-criteria optimization and represent input values for the VIKOR method. The compromise solution is reached through the following steps of the VIKOR algorithm: determination of ideal solution, transformation of varied criterion functions, determination of the criterion weight \( w_i \), determination of the criterion weight \( v \), determination of the measures \( S_j, R_j, Q_j \), \( j \ldots J \), ranking in terms of the values \( S, R, Q \). The compromise solution is a solution closest to the ideal solution based on the adopted distance to the ideal solution.

Tab. 1. Values of the criterion functions (\( f_1 \) – environmental risks, \( f_2 \) – preservation of other resources, \( f_3 \) – economic criterion, \( f_4 \) – nautical conditions, \( f_5 \) – maritime conditions, \( f_6 \) – position and accessibility, \( f_7 \) – project feasibility and \( f_8 \) – demand of nautical berths)

<table>
<thead>
<tr>
<th>varijante</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( f_4 )</th>
<th>( f_5 )</th>
<th>( f_6 )</th>
<th>( f_7 )</th>
<th>( f_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>max</td>
</tr>
<tr>
<td>A1. Rt Kob.</td>
<td>3</td>
<td>0.1</td>
<td>4,500,000</td>
<td>50</td>
<td>58</td>
<td>58</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>A2. Njivice</td>
<td>3</td>
<td>1.8</td>
<td>650,000</td>
<td>40</td>
<td>59.1</td>
<td>55</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>A3. Meljine</td>
<td>2</td>
<td>0.32</td>
<td>700,000</td>
<td>30</td>
<td>59.1</td>
<td>50</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A4. Zelenika</td>
<td>2</td>
<td>0.43</td>
<td>600,000</td>
<td>30</td>
<td>59.1</td>
<td>47</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A5. Kumbor</td>
<td>3</td>
<td>0.23</td>
<td>7,200,000</td>
<td>30</td>
<td>59.1</td>
<td>44</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A6. Kotor</td>
<td>1</td>
<td>0.32</td>
<td>3,500,000</td>
<td>30</td>
<td>57</td>
<td>9</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>A7. Prcanj</td>
<td>1</td>
<td>2.82</td>
<td>655,000</td>
<td>30</td>
<td>57</td>
<td>13</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A8. Bonici</td>
<td>2</td>
<td>0.1</td>
<td>620,000</td>
<td>30</td>
<td>55</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A9. Bigovo</td>
<td>3</td>
<td>0.04</td>
<td>4,200,000</td>
<td>40</td>
<td>54</td>
<td>10</td>
<td>2</td>
<td>2</td>
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<tr>
<td>A10. Budva</td>
<td>2</td>
<td>2.19</td>
<td>6,100,000</td>
<td>30</td>
<td>52</td>
<td>23</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>A11. Sv. Nikola</td>
<td>2</td>
<td>0.57</td>
<td>730,000</td>
<td>30</td>
<td>52</td>
<td>24</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A12. Buljar.</td>
<td>3</td>
<td>2.35</td>
<td>2,100,000</td>
<td>40</td>
<td>22</td>
<td>43</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>A13. Bar</td>
<td>3</td>
<td>1.62</td>
<td>20,100,000</td>
<td>20</td>
<td>0.0</td>
<td>61</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>A14. Liman</td>
<td>2</td>
<td>0.1</td>
<td>8,800,000</td>
<td>40</td>
<td>0.0</td>
<td>87</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A15. Port Milena</td>
<td>2</td>
<td>10.1</td>
<td>580,000</td>
<td>40</td>
<td>0.0</td>
<td>90</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A16. Ada Bojana</td>
<td>2</td>
<td>2.75</td>
<td>1,100,000</td>
<td>50</td>
<td>0.0</td>
<td>102</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Tab. 2. Outputs of the multi-criteria ranking (\( W(I) \)- the first ranking process, \( W(II) \)- the second ranking process and \( W(III) \)- the third ranking process)

<table>
<thead>
<tr>
<th>R.L.</th>
<th>W(I) (0.125 0.125 0.125 0.125 0.125 0.125 0.125 0.125)</th>
<th>W(II) (0.167 0.167 0.167 0.083 0.083 0.083 0.167 0.083)</th>
<th>W(III) (0.083 0.083 0.083 0.167 0.167 0.167 0.083 0.167)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QR</td>
<td>Q</td>
<td>QS</td>
<td>QR</td>
</tr>
<tr>
<td>1.</td>
<td>A10</td>
<td>A10</td>
<td>A6</td>
</tr>
<tr>
<td>3.</td>
<td>A3</td>
<td>A2</td>
<td>A8</td>
</tr>
<tr>
<td>4.</td>
<td>A4</td>
<td>A5</td>
<td>A2</td>
</tr>
<tr>
<td>5.</td>
<td>A5</td>
<td>A11</td>
<td>A5</td>
</tr>
<tr>
<td>6.</td>
<td>A8</td>
<td>A4</td>
<td>A11</td>
</tr>
<tr>
<td>7.</td>
<td>A11</td>
<td>A3</td>
<td>A4</td>
</tr>
<tr>
<td>9.</td>
<td>A1</td>
<td>A1</td>
<td>A14</td>
</tr>
<tr>
<td>10.</td>
<td>A7</td>
<td>A7</td>
<td>A7</td>
</tr>
<tr>
<td>11.</td>
<td>A9</td>
<td>A9</td>
<td>A9</td>
</tr>
<tr>
<td>13.</td>
<td>A12</td>
<td>A12</td>
<td>A9</td>
</tr>
<tr>
<td>15.</td>
<td>A16</td>
<td>A16</td>
<td>A16</td>
</tr>
</tbody>
</table>
introduced for all criterion functions (\( w_i = 0.125 \)). The weight of the decision-making strategy for the majority of criteria is \( v = 0.5 \). In the second ranking process the preference is given to the criteria \( f_1, f_2, f_3 \) and \( f_7 \), while the third ranking process gives preference to the criteria \( f_4, f_5, f_6 \) and \( f_8 \).

In the first ranking, the compromise solution for decision-making is the alternative A10 with 17% of advantage compared to the alternative A8. According to the outputs of the second ranking, the compromise solutions are the alternatives A2, A10, A8 and A5 with the advantage of 1.1%, 1.4%, 0.8%, and 3.7%, respectively. The compromise solutions within the third ranking are the alternatives A10 and A6 with the advantage of 9.6% and 16%, respectively.

CONCLUSION

Proposal of optimum locations for marina construction on the Montenegrin Coast is determined based on the analysis of the outputs listed in Tab. 2. Additionally, consideration should be given to the type of marina e.g. whether the marina is classified as service, standard, small or marina intended for commercial berths. Among service marinas, the alternative A13 (Bar) is at the same time the only marina of this type taken into consideration for multi-criteria optimization, given that the service marina with all the required facilities was constructed in Tivat in 2010. Among standard marinas, the alternatives A5 (Kumbor) and A1 (Rt Kobila) are well ranked, while the alternatives A9 (Bigovo) and A14 (Liman in Ulcinj) rank near bottom of the rank list. Among small marinas, the alternative A10 (Budva) is best ranked. The alternative A6 (Kotor) is well ranked in the first and third ranking, yet it is ranked worst in the second ranking with the 11.6% lag compared to the alternative A16 (Ada Bojana). In the group of commercial berths, the best ranked alternatives are A8 (Bonici), A2 (Njivice), A11 (Sv. Nikola), A4 (Zelenika) and A3 (Meljine). Based on the finite set of compromise solutions in these three rankings, it can be ascertained that the best locations among the total set are A13 (Bar) for service marina, A5 (Kumbor) for standard marina, A10 (Budva) among small marinas and A8 (Bonici) among commercial berths. The obtained results can be a useful tool for a decision maker in defining the order of marina construction on the Montenegrin Coast. Therefore, identification of the micro-location for marina construction should be based on a modern approach which harmonizes all objectives expressed in terms of the criterion functions and leads to determination of the compromise solution. The compromise solution derived from multi-criteria optimization is only a proposal, while a final decision remains with a decision maker.

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The Ship Handling Research and Training Centre at Ilawa is owned by the Foundation for Safety of Navigation and Environment Protection, which is a joint venture between the Gdynia Maritime University, the Gdansk University of Technology and the City of Ilawa.

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is the oldest and largest scientific and technological academic institution in the Pomeranian region. The history of Gdansk University of Technology is marked by two basic dates, namely: October 6, 1904 and May 24, 1945.

The first date is connected with the beginning of the technical education at academic level in Gdansk. The second date is connected with establishing of Gdansk University of Technology, Polish state academic university. Gdansk University of Technology employ 2,500 staff, 1,200 whom are academics. The number of students approximates 20,000, most of them studying full-time. Their career choices vary from Architecture to Business and Management, from Mathematics and Computer Science to Biotechnology and Environmental Engineering, from Applied Chemistry to Geodesics and Transport, from Ocean Engineering to Mechanical Engineering and Ship Technology, from Civil Engineering to Telecommunication, Electrical and Control Engineering. Their life goals, however, are much the same - to meet the challenge of the changing world. The educational opportunities offered by our faculties are much wider than those of other Polish Technical universities, and the scientific research areas include all of 21st Century technology. We are one of the best schools in Poland and one of the best known schools in Europe - one that educates specialists excelling in the programming technology and computer methods used in solving complicated scientific, engineering, organizational and economic problems.

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The Faculty of Ocean Engineering and Ship Technology (FOEST) as the only faculty in Poland since the beginning of 1945 has continuously been educating engineers and doctors in the field of Naval Architecture and Marine Technology.

The educational and training activities of FOEST are supported by cooperation with Polish and foreign universities, membership in different international organizations and associations, as well as participation in scientific conferences and symposia. Hosting young scientists and students from different countries is also a usual practice in FOEST.

The activities of Faculty departments are related to: mechanics and strength of structures, hydromechanics, manufacturing, materials and system quality, power plants, equipment and systems of automatic control, mostly in shipbuilding, marine engineering and energetic systems.

FOEST is a member of such organizations like WEGEMT; The Association of Polish Maritime Industries and the co-operation between Nordic Maritime Universities and Det Norske Veritas. The intensive teaching is complemented and supported by extensive research activities, the core of which is performed in close collaboration between FOEST staff and industry. We take great care to ensure that the applied research meet both the long term and short term needs of Polish maritime industry. FOEST collaborates with almost all Polish shipyards. Close links are maintained with other research organizations and research institutions supporting the Polish maritime industry, such as Ship Design and Research Centre and Polish Register of Shipping, where several members of the Faculty are also members of the Technical Board.

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